Efficiency of a market economy - economy 2 - Dynamic Economy with Single Type of Consumer

Let's take a look at an economy in which consumers choose how much to consume when the time horizon exceeds a single period. For simplicity, we'll consider two periods. Thus, consumers choose consumption today which will call period 1, savings today, which will be used to increase the capital stock in period 2, and consumption for period 2.

The economy operates over for two periods. There is a constant returns to scale technology that uses capital and labor to produce output. The technology is operated by a firm that behaves competitively. Some output is consumed, and some is saved to augment the stock of capital. Note at the end of period 2, there is no need to save, as there are no future periods. Therefore, consumers will be able to eat all the output in period 2 plus we will also assume that the capital stock can be costlessly converted into consumption. Thus, the consumers will consume all of the available resources in period 2.

Capital earns the rental price $r$, and labor earns the rental price $w$. This is a competitive economy, which means that for this particular economy that the decisions of consumers and firms have no impact on the prices $r$ and $w$. The price of output is normalized to 1.

There are two periods, period 1 and 2. There is a given amount of capital, $k_1$ for period 1 which is already in place when consumers make their period 1 decisions. The difference in capital between period 1 and period 2 will depend on how much consumers save. In this first example, we simplify the problem by assuming that capital does not depreciate. Consumers will choose how much to work in the two periods, and how much to consume and save.

The following preferences simplify the problem:

\[
\max \{ \ln(c_1) - A l_1 + \ln(c_2) - A l_2 \} \tag{1}
\]

Utility is maximized subject to a period 1 and period 2 budget constraint. The period 1 constraint is given by:

\[
(1 + r_1)k_1 + w_1 l_1 \geq c_1 + k_2 \tag{2}
\]

Note that the resources available include the the capital stock, $k_1$, as well as the rental price the consumer receives for allowing the firm to use the capital in production.
Note that in the second period that the consumer can consume the output plus the capital, since there is not a third period. Thus, there is no incentive to save in the second period.

Since consumers always prefer more to less, then the two budget constraints will hold with equality. We therefore can replace the weak inequality in these two constraints with an equality sign.

The firm acts competitively - which means that it’s decisions to do not impact the prices in the economy - and maximizes profits in both periods:

$$\max F(k_1, l_1) - r_1 k_1 - w_1 l_1$$

$$\max F(k_2, l_2) - r_2 k_2 - w_2 l_2$$

Solving for the competitive equilibrium allocations means solving for the efficiency conditions for the firm and the households. The firm’s first order conditions are:

$$F_k(k_1, l_1) = r_1, F_l(k_1, l_1) = w_1$$

$$F_k(k_2, l_2) = r_2, F_l(k_2, l_2) = w_2$$

We have a theory of the pricing of the factors of production. Those prices are equated to their marginal productivities.

The efficiency conditions for the household are solved by either setting up a Lagrangian or substituting the budget constraints into the maximization problem. Here, we set it up by using a Lagrangian. Note that we have two budget constraints for the consumer, one for period one and for period two. $\lambda_1$ will be the Lagrange multiplier on the period 1 budget constraint and $\lambda_2$ will be the Lagrange multiplier on the period 2 budget constraint.

$$L = \max\{\ln(c_1) - A l_1 + \ln(c_2) - A l_2\} + \lambda_1 [(1 + r_1) k_1 + w_1 l_1 - c_1 - k_2] + \lambda_2 [(1 + r_2) k_2 + w_2 l_2 - c_2]$$

Differentiate this function with respect to the five choice variables: $c_1, l_1, k_2, c_2, l_2$

$$\frac{1}{c_1} - \lambda_1 = 0$$
\[-A + \lambda_1 w_1 = 0 \quad (10)\]
\[-\lambda_1 + \lambda_2 (1 + r_2) = 0 \quad (11)\]
\[\frac{1}{c_2} - \lambda_2 = 0 \quad (12)\]
\[-A + \lambda_2 w_2 = 0 \quad (13)\]

We also have the two budget constraints:
\[
(1 + r_1) k_1 + w_1 l_1 - c_1 - k_2 = 0 \quad (14)
\]
\[
(1 + r_2) k_2 + w_2 l_2 - c_2 = 0 \quad (15)
\]

Since $\frac{1}{c_1} = \lambda_1$ and $\frac{1}{c_2} = \lambda_2$, we can simplify these equations as follows by substituting out for $\lambda_1$ and $\lambda_2$:
\[
-A + \frac{w_1}{c_1} = 0 \quad (16)
\]
\[
-A + \frac{w_2}{c_2} = 0 \quad (17)
\]
\[-\frac{1}{c_1} + \frac{1 + r_2}{c_2} = 0 \quad (18)
\]
\[
(1 + r_1) k_1 + w_1 l_1 - c_1 - k_2 = 0 \quad (19)
\]
\[
(1 + r_2) k_2 + w_2 l_2 - c_2 = 0 \quad (20)
\]

Now, recall that the factor prices are just equal to marginal productivities, so we can substitute out for the variables $w_1, r_1, w_2, r_2$ as follows:

\[
-A + \frac{F_1(k_1, l_1)}{c_1} = 0 \quad (21)
\]
\[
-A + \frac{F_1(k_2, l_2)}{c_2} = 0 \quad (22)
\]
\[-\frac{1}{c_1} + \frac{1 + F_2(k_2, l_2)}{c_2} = 0 \quad (23)
\]
$$\begin{align*}
(1 + F_k(k_1, l_1))k_1 + F_l(k_1, l_1)l_1 - c_1 - k_2 &= 0 \\
(1 + F_k(k_2, l_2))k_2 + F_l(k_2, l_2)l_2 - c_2 &= 0
\end{align*}$$

Note that we have 5 unknowns - $c_1, c_2, l_1, l_2$, and $k_2$. We have 5 equations, so we can solve for the 5 endogenous variables.

We will next show that these allocations are Pareto Optimal.

To do this, we solve for the equations that govern allocations for the social optimum problem. This is done by solving the social planner’s problem. A social planning problem maximizes society’s utility subject to the resource constraints. We will see that the first order conditions in the competitive equilibrium problem are the same as in the social optimum problem.

The Social Planning problem requires that we specify utility for society. In this case, society consists of many households that are the same. We therefore will place equal weight on each household’s utility, which means that we can maximize utility for a representative consumer.

$$L = \max \{ \ln(c_1) - A l_1 + \ln(c_2) - A l_2 + \lambda_1 [F(k_1, l_1) + k_1 - c_1 - k_2] + \lambda_2 [F(k_2, l_2) + k_2 - c_2] \}$$

(26)

In the social optimum Lagrangian, we have the utility function for both periods, along with the economy’s resource constraint in the first period, which is multiplied by $\lambda_1$, and the economy’s resource constraint in the second period, which is multiplied by $\lambda_2$. The first order conditions are

$$\frac{1}{c_1} - \lambda_1 = 0$$

(27)

$$-A + \lambda_1 F_l(k_1, l_1) = 0$$

(28)

$$-\lambda_1 + \lambda_2 [1 + F_k(k_2)] = 0$$

(29)

$$\frac{1}{c_2} - \lambda_2 = 0$$

(30)

$$-A + \lambda_2 F_l(k_2, l_2) = 0$$

(31)
We also have the two resource constraints:

\[ F(k_1, l_1) + k_1 - c_1 - k_2 = 0 \]  
\[ F(k_2, l_2) + k_2 - c_2 = 0 \]

As before, we can substitute out for the Lagrange multipliers \( \lambda_1 \) and \( \lambda_2 \). We get the following equations once we make those substitutions:

\[ -A + \frac{F_l(k_1, l_1)}{c_1} = 0 \]  
\[ -A + \frac{F_l(k_2, l_2)}{c_2} = 0 \]  
\[ \frac{1}{c_1} + \frac{1 + F_k(k_2, l_2)}{c_2} = 0 \]  
\[ F(k_1, l_1) + k_1 - c_1 - k_2 = 0 \]  
\[ F(k_2, l_2) + k_2 - c_2 = 0 \]

Note that the first three equations here are identical to the first three equations in the competitive equilibrium problem. The last two equations here are also the same as the last two equations in the competitive equilibrium problem. To see this, compare the period 1 budget constraint in the competitive equilibrium problem with the period 1 resource constraint in the social optimum problem:

\[ (1 + F_k(k_1, l_1))k_1 + F_l(k_1, l_1)l_1 - c_1 - k_2 = 0 \]  
\[ F(k_1, l_1) + k_1 - c_1 - k_2 = 0 \]

Assumptions that are required for this result include: perfect competition, consumers prefer more goods to less goods (this is called non-satiation), and the production function is constant returns to scale (decreasing returns to scale is ok also). The bottom line is that under these assumption, a market economy - with its price system - allocates goods and services such that no one can be
made better off without making someone worse off. In this case, there is a single consumer who consumes over two periods. Thus, we can’t rearrange resources between the first period and the second period to make the consumer better off.

We will later see that the coincidence of a social optimum and the competitive equilibrium does not hold when we make some adjustments to the economy, such as including income taxes.