Notes on Taxes and Government Spending, and the Impact on Economic Activity

We will analyze two types of government spending: purchases of goods and services, which range from national defense expenditures to parks, to roads and bridges, and transfer payments, which is the act of the government transferring income from one person to another. Social security and Medicare are examples of transfer payments. Most of the federal government budget is transfer payments.

To pay for government purchases or transfer payments, the government needs revenue. We will analyze different types of taxes: taxes on labor income, taxes on capital income, sales taxes, and lump sum taxes, which we will define below.

The impact of government spending and taxation on the economy depends on the type of spending, and how it is financed.

First, let’s look at the simplest case, which is the case of government spending is national defense expenditures, and they are financed with lump sum taxes.

A lump sum tax is a tax that is levied on each individual (or household), and the tax liability that the individual (or household) incurs doesn’t depend on the individual’s characteristics. That is, the amount of the tax that must be paid doesn’t depend on income, or the level of consumption, or wealth, or any other possible attribute of the taxpayer. Everybody owes the same tax. Lump sum taxes are sometimes called "head taxes", because it is a tax that is identically levied on each tax payer.

When we talk about national defense spending, we will assume that this is output purchased by the government that doesn’t impact the household’s utility, nor does it impact the production technology. The only affect it has is that it takes away resources from the private sector. Note that we will call this national defense, but it could be other government purchases that don’t affect marginal utility or the production technology.

Example 1 - National Defense Spending with Lump Sum Taxes

There is one type of household. To make this as simple as possible, we do not consider capital in the production function.

The preferences are
max \( u(c) - v(h) \) \hspace{1cm} (1)

The consumer’s budget constraint is

\[ wh = c + T \] \hspace{1cm} (2)

Government spending is exogenous. The government’s budget constraint is:

\[ G = T \] \hspace{1cm} (3)

The economy’s resource constraint is

\[ Y = C + G \] \hspace{1cm} (4)

The firm’s profit maximization problem is

\[ \text{max} \ Ah - \ wh \] \hspace{1cm} (5)

The first order condition for labor and consumption are the same as ones we have worked with before and equate the marginal cost of working to the marginal benefit of working:

\[ v'(h) = u'(c)w \] \hspace{1cm} (6)

The firm’s profit maximization first order condition equates the wage rate to the marginal product of labor:

\[ F'(h) = A = w \] \hspace{1cm} (7)

This means we can combine those two equations to get:

\[ v'(h) = u'(c) * F'(h) = u'(c) * A \] \hspace{1cm} (8)

This last equation, along with the fact that \( Y = Ah = c + G \) gives us two equations in the two unknowns, \( h \) and \( c \).
We can see how changes in government spending impact the economy if we specify the utility function. Suppose that the utility function is \( \ln(c) - \frac{h^2}{2} \).

Then we have for the first order condition for labor:

\[
h = \frac{A}{wh - T} = \frac{A}{Ah - T} \tag{9}
\]

Now let’s vary taxes (which is the same thing as varying government spending) to see what happens to hours worked, consumption, and output. Note that as \( T \) gets large, the marginal utility of consumption rises, all other things equal. This will lead the household to increase the amount of hours that they work compared to \( T \) that is small.

Why do taxes lead to more work? This is because taxes are lump sum. The tax doesn’t change the incentive to work, as a labor income tax would change that incentive (and we will see how that works a bit later).

Specifically, if government is going to take a lot of output, then there will be little left for private consumption, and that means that the marginal utility of consumption rises. This higher marginal utility motivates the household to work more. Economists call this effect the *income effect*. That is, taxes reduce your income, and this leads the household to work more. In the next section, we will introduce the *substitution effect* of taxes. This substitution effect doesn’t appear here because taxes don’t change the return to working.

Now we will show that this economy is Pareto optimal. To see this, we solve the social optimum program, which maximizes utility subject to the constraints, which in this case is the resource constraint, which divides output between the consumer and the government. (Recall that government will be taking an exogenous amount of output). Moreover, recall that with the social optimum, there is no firm maximization problem, no household maximization problem.

\[
\max u(c) - v(h) \tag{10}
\]

subject to the resource constraint

\[
Ah = c + G \tag{11}
\]

The first order conditions for consumption and labor are exactly the same as above:

\[
v'(h) = u'(c) * A \tag{12}
\]
and we have the resource constraint, \( Y = c + G \).

We have two equations in the two unknowns \( c \) and \( h \), and the equations are exactly the same as above. Therefore the values of \( h \) and \( c \) that solve the market equilibrium also solve the social optimum problem, which means that the market economy is efficient - it is Pareto optimal.

**Example 2 - Transfer Payments (or Government Spending that substitutes for private spending) Financed with Labor Income Taxes**

Now we will work with labor income taxes, in which your labor income will be taxed at the tax rate, \( \tau \), \( 0 \leq \tau \leq 1 \).

This tax rate on labor income will change the return to working by reducing the after-tax pay that you receive. We will see that this will impact how much work the household does. Before doing that, we will discuss the income and substitution effects of taxes on labor supply. Suppose we have the utility function \( u(c) - v(h) \). Suppose also that your wage is taxed at rate \( \tau \).

We have

\[
\max u(c) - v(h) \tag{13}
\]

subject to

\[
(1 - \tau)wh + T = c \tag{14}
\]

Note that the budget constraint includes two sources of resources. The first is after-tax labor income, which is \((1 - \tau)wh\). The second is the transfer payment from the government. Specifically, the government collects tax revenue, which is \(\tau wh\), and then gives it back to you. However, the transfer is viewed as exogenous by the household. Thus, we call the exogenous transfer "\( T \)."

The government budget constraint is given by

\[
T = \tau wh \tag{15}
\]

Since there are no government expenditures, the resource constraint is given by:

\[
Y = C \tag{16}
\]
Now, we can see how taxes impact the decision to work. The usual condition that equates the marginal cost of work with the marginal benefit of work changes, because you don’t keep the whole wage, you just keep the after-tax wage. So the equation is given by:

\[ v'(h) = u'(c)(1 - \tau)w \]  

(17)

rather than

\[ v'(h) = u'(c)w \]  

(18)

which is what we have had in this course prior to studying labor income taxes.

Aside - Note that we will get the same result if the government uses the tax revenue to purchases consumption goods for households that households value, such as school lunch programs or other government-provided food, government-provided health care, government-provided recreation facilities, etc. In this case, we modify the consumers utility function as \( U(c + G) - v(h) \), where \( c \) is private consumer spending and \( G \) is spending for consumers done by the government. Now, substitute the household budget constraint into the utility function to get \( U((1 - \tau)wh + G) \). Next, recall that \( G = \tau wh \). This means that the utility function, with these substitutions, becomes: \( U((1 - \tau)wh + \tau wh) = U(wh) \).

We can now talk about income effects of taxes and substitution effects of taxes. We can see this in the consumers first order condition. First note that taxes reduce the incentive to work by directly reducing your take home pay. The higher the tax, the lower is the take home pay: \((1 - \tau)w\), which is a disincentive to work. However, as we saw in the example above, taxes can reduce the resources available to the private section, which raises the marginal utility of consumption. This factor, tends to raise the incentive to work. We call these two effects the substitution and income effects of taxes, respectively.

In this particular case of a labor income tax, and the tax revenue is used only as a transfer payment, there is only a substitution effect of taxes. To see this, substitute the budget constraint into the first order condition. Recall the budget constraint is \((1 - \tau)wh + T = c\), and therefore the first order condition becomes:

\[ v'(h) = u'((1 - \tau)wh + T) * (1 - \tau)w \]  

(19)

Next, note that the transfer payment, \( T \), is equal to the tax revenue collected, \( \tau wh \). This means we have:
\[ v'(h) = u'((1 - \tau)wh + \tau wh) \ast (1 - \tau)w \]  

(20)

Combining the terms \(-\tau wh\) and \(\tau wh\), we get

\[ v'(h) = u'(wh) \ast (1 - \tau)w \]  

(21)

In this case when the government is not taking any resources, so the marginal utility of consumption is not impacted by any government decisions, because the tax revenue is transfered back to households, the tax decision doesn’t impact the consumer’s income or marginal utility, as the household consumes all of output. The income effect is therefore zero in this case. We therefore only have the substitution effect.

Next, note that the size of the substitution effect is determined the marginal cost of working, \(v'(h)\).

To see this, consider the function form for \(v, \frac{h^\alpha}{\tau} \), where \(\alpha > 1\). This functional form is a standard one that is used in the analysis of taxation. We can now see how the value of the coefficient \(\alpha\) is involved with how much taxes influence hours worked. To see this, note that in this case, \(v'(h)\) is \(h^{\alpha - 1}\).

The income effect means that taxes reduce the household income, and we see this when we substitute out for consumption in the marginal utility of consumption. This means that all other things equal, taxes increase the marginal utility of consumption, which motivates you to work harder. The substitution effect means that the return to working is impacted by taxes, as you keep \((1 - \tau)w\), rather than keeping \(w\). To isolate the impact of the substitution effect, we will assume that all tax revenue is redistributed back to the household, as discussed above. Since we will be holding consumption constant, we will indicate that with an overbar:

\[ h^{\alpha - 1} = \overline{u'(wh)} \ast (1 - \tau)w \]  

(22)

Next, isolate the term \(h\) on the left hand side by exponentiating by \(\frac{1}{\alpha - 1}\):

\[ h = \left(\overline{u'(wh)} \ast (1 - \tau)w\right)^{\frac{1}{\alpha - 1}} \]  

(23)

Now, let’s differentiate \(h\) with respect to \((1 - \tau)w\), which allows us to calculate the impact of a change in the after-tax wage on hours.
This seems complicated, but let’s use that expression to calculate the elasticity of hours with respect to after-tax wages, which is much simpler, and we do this using the normal formula to construct an elasticity. Recall that the elasticity of the variable $y$ with respect to $x$ is given by

$$\frac{dy}{dx} \cdot \frac{x}{y}$$

So the elasticity of hours worked with respect to after-tax income, is just determined by the coefficient $\alpha$. Thus, high values of $\alpha$ mean that the elasticity is low, which means that changes in after-tax wages will have a small effect on hours worked. A reasonable range for this coefficient is around 3. This means that a 1 percent change in the after-tax wage rate will result in around a 1/2 percent change in hours worked.

The paper by Ohanian, Raffo, and Rogerson looks at historical evidence on how taxes changed across countries and across time, and uses a model similar to this one to evaluate how much of the change in hours worked in different countries is due to taxes. We will discuss this paper in class.

**Case 3 - Military Spending Financed with Labor Income Taxes - World War II**

World War II is a good case to analyze the impact of government spending and taxes on the economy when resources are devoted to military spending, and the war effort was financed in part by income taxes. During World War II, military spending rose dramatically, increasing by nearly 100-fold between the mid-1930s to 1944. In fact, government spending at the peak of the war in 1944 was more than total GNP in 1939! You can see from this statistic that there was a huge income effect. Tax rates also rose, with labor income taxes rising from about 12 percent to about 20 percent. To see how this impacted consumer behavior, suppose the utility function was given by:

$$\ln(c) - \phi \frac{h^2}{2}$$

Suppose that the production function and resource constraint is given by $Ah = c + G$.

The first order condition for the consumer (note that we have derived this condition many times before) is:
\[ \phi h = \frac{1}{c} (1 - \tau) w \] 

(27)

or

\[ \phi h = \frac{1}{c} (1 - \tau) A \] 

(28)

Now, let’s see how the substitution effect and the income effect impacted decisions. First, let’s look at the substitution effect. Analyzing the substitution effect means we look at the impact of taxes, but hold consumption constant. The tax rates rise from 12 percent to 20 percent. This means that the after-tax wage, \((1 - \tau)w\), falls by about 10 percent (from \(0.88w\) to \(0.8w\)). Thus, the substitution effect implies that hours worked would also have to fall by about 10 percent, given that we are holding consumption constant.

Now, consider the income effect of the government spending. Recall that government spending alone would take more than all of the output produced! This means that households will need to work considerably more to produce enough output for the government to fight the war as well as provide goods for private uses. Thus, we can see in this case that the income effect of the war dominates the substitution effect. Specifically, the substitution effect depresses labor, but the remarkable increase in government spending motivates people to work more.

Case 4: Government Investment and Taxes

In addition to transfers and military spending, government also does investment projects, such as roads, bridges, and other types of infrastructure. A well-known investment project was the Interstate Highway System, which was started in the 1950s, and continues to today. The idea was to provide high speed roadways that would link the country and enhance business transactions, reduce travel time, and expand commerce.

Note that these investments tend to increase the productivity of labor and private capital. We can therefore augment our production function to incorporate government-provided capital. Recall the production function we have used before was

\[ Y = AK^{1/3}L^{2/3} \] 

(29)

Now, with government capital, we have

\[ Y = A^*K^{1/3}L^{2/3} \] 

(30)
where the productivity coefficient \( A^* \) is given by: \( A^* = AG^\alpha \), where \( G \) is government capital, and \( \alpha \) is a coefficient that controls the impact of government capital in production. The coefficient has been estimated to be about 0.1.

To see how government provided capital increases the productivity of private and capital and labor, note that the marginal products of capital and labor, respectively,

\[
\frac{1}{3} AG^\alpha \left( \frac{L}{K} \right)^{2/3} \tag{31}
\]

\[
\frac{2}{3} AG^\alpha \left( \frac{K}{L} \right)^{1/3} \tag{32}
\]

Note how a higher level of government capital, \( G \), raises the productivity of both capital and labor. In terms of capital, you can see how highways allow trucks (capital) to travel faster, and you can see how drivers (labor) are more productive as well.