Notes on Start-ups and Their Impact on the Economy

These notes will show how start-ups (new businesses) contribute to economic growth. The start-up will have higher productivity, and we will see how the start-up will draw resources away from the incumbent business. The start-up will result in higher output and consumption.

There is a single type of household with preferences over consumption and working. There are two firms, which are competitive, and they have the same technology to produce goods, which is $A h_i^{1/2}$, where $i$ indexes the firm. The social optimum is to maximize utility subject to the single resource constraint. We begin with two production locations (businesses) that have the same productivity, then we will replace one of the businesses with a start-up.

We maximize social utility:

$$\max \{ c - \phi \frac{(h_1 + h_2)^2}{2} \}$$  \hspace{1cm} (1)

subject to the resource constraint, which

$$Ah_1^{1/2} + Ah_2^{1/2} = c$$  \hspace{1cm} (2)

The Lagrangian is

$$L = \max \{ c - \phi \frac{(h_1 + h_2)^2}{2} + \lambda (Ah_1^{1/2} + Ah_2^{1/2} - c) \}$$  \hspace{1cm} (3)

The first order conditions are

$$1 = \lambda$$  \hspace{1cm} (4)

$$\phi(h_1 + h_2) = \lambda A h_1^{-1/2} = \lambda A h_2^{-1/2}$$  \hspace{1cm} (5)

and the resource constraint:

$$Ah_1^{1/2} + Ah_2^{1/2} = c$$  \hspace{1cm} (6)
The first order conditions imply that $h_1 = h_2 = h/2$, so both locations employ the same amount of people and produce the same amount of output: 1/2 of the total. With a bit of algebra, and assuming that $A = 1$, we find that $h = \left(\frac{1}{2}\right)^{\frac{1}{3}} \cdot \phi^{-\frac{2}{3}} = .79 \cdot \phi^{-\frac{2}{3}}$

Now suppose that one of the production locations in the economy, location 2, is taken over by a startup, which has a higher level of productivity, $A^H$. Let’s see what happens to the economy with the startup that has higher productivity.

Preferences remain the same, but the resource constraint becomes:

$$Ah_1^{1/2} + A^H h_2^{1/2} = c$$  \hspace{1cm} (8)

The social optimum first order conditions are

$$1 = \lambda$$  \hspace{1cm} (9)

$$\phi(h_1 + h_2) = \frac{A}{2} h_1^{-1/2} = \frac{A^H}{2} h_2^{-1/2}$$  \hspace{1cm} (10)

We can rearrange these terms as follows:

$$h_1 = \left(\frac{A}{A^H}\right)^2 h_2$$  \hspace{1cm} (11)

Plugging this back into the equation $\phi(h_1 + h_2) = \frac{A}{2} h_1^{-1/2}$, we get

$$h_2 = \left(\frac{A^H}{2 \left(1 + \left(\frac{A}{A^H}\right)^2\right)}\right)^{2/3}$$  \hspace{1cm} (12)

Recall that when $A^H = A = 1$, then $h_2 = h_1 = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{1/3} \cdot \phi^{-2/3} = \frac{.79}{2} \cdot \phi^{-2/3}$.

Now suppose that $A^H = 2$, and $A = 1$. Now we get $h_2 = .86 \cdot \phi^{-2/3}$, and $h_1 = .22 \cdot \phi^{-2/3}$. Thus, there is a tremendous reallocation of resources (labor) away from the first plant to the second plant. Specifically, labor at the first plant falls by about 46 percent, and labor at the second plant rises by 117 percent.
The reason that the first plant contracts so much is because it is less productive than the second plant: $A < A^H$. From society's perspective, resources should be allocated so that society extracts maximum output from the marginal unit. This occurs when the marginal product of labor at the two plants is equated, which is the condition that $A^H h_2^{-1/2} = A^H h_2^{-1/2}$.

Note that society is better off with the start-up, as it has higher productivity. As the start-up draws resources away from the incumbent, the incumbent business shrinks. Economists call this process "creative destruction". The start-up is the "creation". As that creation grows, it siphons off resources from incumbents, and we call that part "destruction".