The problem set is intended to give you a comprehensive review of exam-grade material on school choice. The problem set is optional and ungraded. It will be discussed in sections, and the solutions will be posted. You may rely on all results derived in class notes without re-proving any of them.

Consider an economy with 4 students $s_1, ..., s_4$ and 4 schools $A, B, C, D$. The preferences of the students are as follows:

- $s_1: A > B > C > D$
- $s_2: C > B > D$
- $s_3: B > C > D$
- $s_4: C > B > D$

Suppose that students’ neighborhood schools as follows: $s_1$ neighborhood school is school $A$, student $s_2$ neighborhood school is $B$, student $s_3$ neighborhood school is $C$, and student $s_4$ neighborhood school is $D$.

Questions

1. Consider the allocation $\{(s_1, A), (s_2, C), (s_3, D), (s_4, B)\}$. Is it Pareto efficient? Why or why not?

2. For this question only, assume that the school seats are allocated to students via a serial dictatorship in which student $s_1$ picks first, then student $s_2$, then $s_3$, and finally $s_4$.
   (a) What is the resulting assignment of school seats to students?
   (b) Is the assignment Pareto efficient? Why, or why not?
   (c) Let us assume that all students but possibly student $s_2$ submit their true preference rankings. Can student $s_2$ get assigned a better school seat by submitting a preference ranking different from his true preference ranking? If yes, please provide such an improving preference ranking. If not, please provide an argument why not.
   (d) Is every student receiving an assignment as good as their neighborhood school?

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1How to read this table? Each row gives preference ranking of a student, listing schools from the student most preferred school all the way to his least preferred but still acceptable school (schools which are not listed are not acceptable).
3. For this question only, assume that the school seats are allocated to students via deferred acceptance with students proposing. For the sake of the mechanism, school priorities are as follows:

\[ A : s_1 > s_2 > s_3 > s_4 \]
\[ B : s_2 > s_1 > s_3 > s_4 \]
\[ C : s_3 > s_1 > s_2 > s_4 \]
\[ D : s_4 > s_1 > s_2 > s_3 \]

(a) What is the resulting assignment of school seats to students?
(b) Is the assignment Pareto efficient? Why, or why not?
(c) Let us assume that all students but possibly student \( s_2 \) submit their true preference rankings. Can student \( s_2 \) get assigned a better school seat by submitting a preference ranking different from his true preference ranking? If yes, please provide such an improving preference ranking. If not, please provide an argument why not.
(d) Is every student receiving an assignment as good as their neighborhood school?

4. For this question only, assume that the school seats are allocated to students via a Top-Trading-Cycles mechanism in which student \( s_1 \) is endowed with school \( A \), student \( s_2 \) is endowed with school \( B \), student \( s_3 \) is endowed with school \( C \), and student \( s_4 \) with school \( D \).

(a) What is the resulting assignment of school seats to students? Please describe the run of the TTC algorithm.
(b) Is the assignment Pareto efficient? Why, or why not?
(c) Let us assume that all students but possibly student \( s_1 \) submit their true preference rankings. Can student \( s_1 \) get assigned a better school seat by submitting a preference ranking different from his true preference ranking? If yes, please provide such an improving preference ranking. If not, please provide an argument why not.
(d) Is every student receiving an assignment as good as their neighborhood school?

5. For this question only, assume that the school seats are allocated to students via a Top-Trading-Cycles mechanism in which student \( s_1 \) is endowed with school \( D \), student \( s_2 \) with school \( A \), student \( s_3 \) with \( C \), and student \( s_4 \) with \( B \). Please note that in this TTC students’ endowments are different from their neighborhood schools.

(a) What is the resulting allocation?
(b) Please describe the run of the TTC algorithm.
(c) Is every student receiving an assignment as good as their neighborhood school?

\[ ^2\text{Each row gives priority ordering of a school, starting on the left with the highest priority student.} \]