The problem set is intended to give you a comprehensive review of exam-grade material on school choice. The problem set is optional and ungraded. It will be discussed in sections, and the solutions will be posted. You may rely on all results derived in class notes without re-proving any of them.

Consider an economy with 4 students $s_1, \ldots, s_4$ and 4 schools $A, B, C, D$. The preferences of the students are as follows\(^1\)

- $s_1 : A > B > C > D$
- $s_2 : C > B > D$
- $s_3 : B > C > D$
- $s_4 : C > B > D$

School district sets school priorities are as follows\(^2\)

- $A : s_1 > s_2 > s_3 > s_4$
- $B : s_2 > s_1 > s_3 > s_4$
- $C : s_3 > s_1 > s_2 > s_4$
- $D : s_4 > s_1 > s_2 > s_3$

Suppose that students’ neighborhood schools as follows: $s_1$ neighborhood school is school $A$, student $s_2$ neighborhood school is $B$, student $s_3$ neighborhood school is $C$, and student $s_4$ neighborhood school is $D$.

Notice that the preferences and neighborhood schools are exactly as in Part 1 of the problem set. The priorities are identical to the priorities in question 3 of Part 1. Since Part 1 had questions 1-5, we continue the numbering starting with question 6 in the current part of the problem set.

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\(^1\)How to read this table? Each row gives preference ranking of a student, listing schools from the student most preferred school all the way to his least preferred but still acceptable school (schools which are not listed are not acceptable).

\(^2\)Each row gives priority ordering of a school, starting on the left with the highest priority student.
Questions

6. Consider the outcomes of serial dictatorship, deferred acceptance, and top trading cycles calculated in Part 1 of this problem set.
   (a) Is the outcome of the serial dictatorship stable?
   (b) Is the outcome of the deferred acceptance stable?
   (c) Is the outcome of top trading cycles stable?

7. For this question only, assume that the school seats are allocated to students via the Boston Mechanism.
   (a) What is the resulting assignment? Describe the run of the algorithm.
   (b) Is it efficient? Why, or why not?
   (c) Is it stable? Why or why not?
   (d) Is every student assigned at least his or her neighborhood school?
   (e) Let us assume that all students but possibly student $s_2$ submit their true preference rankings. Can student $s_2$ get assigned a better school seat by submitting a preference ranking different from his true preference ranking? If yes, please provide such an improving preference ranking. If not, please provide an argument why not.

8. Find a Nash equilibrium of the Boston Mechanism and provide an argument that it is indeed a Nash equilibrium.

9. Consider the Nash equilibrium outcome of the Boston Mechanism you calculated in question 8.
   (a) Is it efficient? Why or why not?
   (b) Is it stable? Why or why not?
   (c) Is every student assigned at least his or her neighborhood school?
   (d) Keeping the Nash equilibrium reports of students $s_1$, $s_2$, and $s_3$ fixed, can student $s_4$ obtain a better outcome than the Nash equilibrium outcome?