SOLUTION TO PROBLEM SET 2: No-Transfer Allocation and Exchange.
(Part 2) School Choice
ECON 106D

Alternative answers and optional explanations in italics. On exams, one answer is of course sufficient.

6. a) b) c) Note that the outcome of three different mechanism is \{(s1, A), (s2, C), (s3, B), (s4,D)\}. By Theorem 1 of lecture 3, an outcome of DA algorithm is stable.

Alternative answer: One can check there is no blocking individual or pair. Note that there is no blocking individual. s1, s2, and s3 are all matched with their top choices, so they cannot be a part of a blocking pair. For the same reason, A and D cannot be a part of a blocking pair. B’s current match is s3 whose priority is higher than s4. C’s current match is s2 whose priority is also higher than s4. Therefore, there is no blocking pair.

7. a) The Boston Mechanism runs as follows:

Round 1. School A has one applicant, s1. School B has one applicant, s3. School C has two applicants, s2 and s4. Each school admits the applicant with highest priority: A admits s1 as the only applicant; B admits s3 as the only applicant; and C admits s2 because s2 has higher priority than s4 at C. Alternatively, we can run round 1 of BM by observing that: s1’s top choice is A, s2’s top choice is C, s3’s top choice is B and s4’s top choice is C. Since s2 has priority at school C over s4 then the following assignments are made: \{(s1, A), (s2, C), (s3, B)\}.

Round 2. D has an open seat but no unmatched student listed it as second choice Alternatively: s4’s second choice is B, but the seat is already filled.

Round 3: D has one applicant who ranked it third, s4. Thus D admits s4. Alternatively: s4’s third choice is D, and the following assignment is made \{(s4,D)\}.

The final assignment is \{(s1, A), (s2, C), (s3, B), (s4,D)\}.

b) The outcome is same to the outcome generated by serial dictatorship or TTC. Therefore, it is efficient. Alternative answer: By Theorem 2 of lecture 10, BM generates an efficient outcome with respect to reported preferences.

c) The outcome is stable, because it is the same outcome as problem 6.

d) s1 and s4 are assigned to the schools in neighborhood and s2 and s3 end up with better schools.

e) No. s2 already got his top choice.

8. The following strategy profile is a Nash equilibrium.
s1: A>B>C>D;  s2: C>B>D;  s3: B>C>D;  s4: D>C>B
The Boston mechanism ends in first round which result in the same outcome \{(s1, A), (s2, C), (s3, B), (s4, D)\}. s1, s2, and s3 are assigned to their top choices, so they are doing their best responses. It is enough to show that s4 cannot get a better school by playing otherwise. If he alters his report without changing the top choice, it won’t make any difference. If he places B or C as his top choice, still he cannot get in those schools because he has the lowest priority in those schools and cannot win the schools over s2 or s3. Therefore, there is no way that s4 gets B or C as long as s2 and s3 place C and B as their top choices.

There is another Nash equilibrium that results in a different outcome.

\begin{align*}
  s1: & A>B>C>D; \\
  s2: & B>C>D; \\
  s3: & C>B>D; \\
  s4: & D>C>B
\end{align*}

Boston mechanism ends in the first round, and the outcome is \{(s1, A), (s2, B), (s3, C), (s4, D)\}.

First of all, s1 gets his first choice, so has no incentive to play otherwise. Think about s2. If he alters his ranking with maintaining his top choice as B, it won’t make any difference in the final outcome. If he places D as his top choice, then he will end up with D which is worse than B for him. If he places C as his top choice, then still he cannot get into C because he cannot compete with s3 whose priority is higher in C. In conclusion, s2 cannot get into C by playing otherwise. The same logic applies for s3, because s2 has a higher priority in B. For s4, he cannot compete with either s2 or s3, so it is not possible to get into a better school for s4 as long as the others play the above strategies.

9. Depending on which equilibrium outcome you use, the answer for following questions can be different. Let’s use the outcome \{(s1, A), (s2, C), (s3, B), (s4, D)\} as a benchmark followed by the alternative answer written in \textit{Italic}.

a) b) c) This outcome is same to the one in previous problems. Therefore, it is efficient, stable, and every students get into at least as good as neighborhood schools.

d) Since the strategy profile is a Nash equilibrium, s4 cannot get into a better school by reporting other rankings as long as the others’ strategies are fixed.

Now, consider the alternative outcome with different Nash equilibrium \{(s1, A), (s2, B), (s3, C), (s4, D)\}

a) The outcome is not efficient, because s2 and s3 can swap their current school and get better outcome for both without hurting s1 and s4. (This is so though BM generates efficient outcome with respect to reported rankings).

b) The outcome is stable. Note that all schools have student with highest priority, so they cannot be a part of a blocking pair. Or, one can verify the outcome is a result of school proposing DA algorithm.

c) All students are assigned to the neighborhood schools.

d) Since the strategy profile is a Nash equilibrium, s4 cannot get into a better school by reporting other rankings as long as the others’ strategies are fixed.