A seller has a single indivisible good for sale. There are three potential bidders; each bidder \( i = 1, 2, 3 \) has value \( v_i \) for the auctioned good. In this problem set we assume that all bids in \([0, \infty)\) are allowed.

**In questions 1-3**, assume that bidders values are \( v_1 = v_2 = v_3 = 3 \), and that each bidder knows everyone else’s values.

1. Construct two different equilibria of the second-price auction. Provide an argument why the profiles of strategies you construct are in equilibrium.

2. Is bidder 1 bidding \( b_1 = 2 \), bidder 2 bidding \( b_2 = 2.5 \), and bidder 3 bidding \( b_3 = 1.5 \) an equilibrium of the second-price auction? Why or why not?

3. Construct an equilibrium of the first-price auction. Provide an argument why the profiles of strategies you construct are in equilibrium.

**In questions 4-7**, assume that each value \( v_i \) is an independent draw from \( \{1, 2\} \) and takes the high value 2 with probability \( \pi = \frac{1}{2} \). Each bidder knows that other bidders’ values are drawn independently from this distribution. The bidder also knows his own value but does not know the realization of values of other bidders.

4. Is bidding the true value a dominant strategy for bidder 1 in the second-price auction? Why or why not?

5. Construct two different equilibria of the second-price auction.

6. Is bidding the true value a dominant strategy for bidder 1 in the first-price auction? Why or why not?

7. Is bidding \( b_i = \frac{1}{2} v_i \) a (pure-strategy) equilibrium in the first-price auction?