Econ 106D. Problem Set on First-Price Auctions with Restricted Bid Spaces.

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The problem set is optional and ungraded. The problems will be discussed in sections, and the solutions will be posted. You may rely on all results derived in class notes without re-proving any of them.

A seller has a single indivisible good for sale. There are three potential bidders; each bidder \( i = 1, 2, 3 \) has value \( v_i \in \{2, 4\} \) for the auctioned good. The values are distributed independently so that the high value \( v_i = 4 \) has probability \( \frac{1}{3} \) and the low value \( v_i = 2 \) has probability \( \frac{2}{3} \).

You may restrict attention to symmetric equilibria in pure strategies. In efficiency questions, assume that the seller’s value for the good is 0. In all questions we are considering first-price auctions.

1. Suppose only two bid levels are allowed \( \ell = 2 \) and \( h = 4 \). Construct an equilibrium of the first-price auction. Provide an argument why the profiles of strategies you construct are in equilibrium. Is the equilibrium efficient? What is the expected revenue?

2. Suppose only two bid levels are allowed \( \ell = 2 \) and \( h = 3 \). Construct an equilibrium of the first-price auction. Provide an argument why the profiles of strategies you construct are in equilibrium. Is the equilibrium efficient? What is the expected revenue?

3. Suppose \( \ell = 2 \). What is the revenue maximizing choice of the high bid \( h \)? What is the resulting revenue? Is the resulting auction efficient?

4. Suppose the seller sells only to high-value bidders that is sets \( \ell = h > 2 \). What is the revenue maximizing value of \( h \)? What is the resulting expected revenue of the seller? Is the resulting auction efficient?

5. Is the seller’s expected revenue higher when the seller sells to both low and high-value bidders (that is sets \( \ell = 2 \) and optimal \( h \) as in question 3) or is it higher when the seller sells only to high-value bidders (that is sets optimally \( \ell = h > 2 \) as in question 4)?