Midterm exam       PS 30       November 2007

Name:

TA:

Section number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has four parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question on Part 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. Please turn in your exam to your TA. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
Part I. Hovannes’ Toys, the largest action figure distributor in Blueland, outsources its manufacturing to the country of Gaoland. However, Hovannes’ Toys recently learned that their action figures had been made with a chemical that is potentially toxic to children. The company now faces the decision of whether to recall over 150,000 of their figures. Due to previous toy safety concerns (not to mention pressure from lobbyists from competing puzzle companies), the president of Blueland is simultaneously considering new legislation that would fine companies who sold defective action figures, whether or not they were recalled. Assume a strategic form game, with payoffs as follows:

<table>
<thead>
<tr>
<th></th>
<th>Recall</th>
<th>Don’t Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulate</td>
<td>10, 10</td>
<td>15, -10</td>
</tr>
<tr>
<td>Don’t Regulate</td>
<td>5, 15</td>
<td>-5, -5</td>
</tr>
</tbody>
</table>

a. (3 points) Find all Pure Strategy Nash Equilibria (no need to find mixed strategy NE).

NE:
b. (6 points) Johann's Toys is another toy distributor, based in Gaoland. Johann's Toys was aware of the potential dangerous chemicals and very carefully avoided them. For obvious reasons, the reputation of Johann's Toys would suffer greatly if Hovannes' Toys pursued their recall. Johann's Toys is simultaneously considering whether or not to threaten Hovannes' Toys with a lawsuit over the potential tarnishing of their image.

Please expand this model by adding Johann's Toys as a third player. Consider these conditions in determining payoffs:

i. The payoffs for Hovannes' Toys and the government are as displayed in the table above, if Johann's Toys does not threaten to sue
ii. If Johann's Toys threatens to sue, the government's payoffs remain as above and the payoffs of Hovannes' Toys are all reduced by 5
iii. If Hovannes' Toys does not recall, Johann's Toys gets a payoff of 0, whether or not they threaten to sue
iv. If Hovannes' Toys recalls, Johann's Toys gets a payoff of -15 if they don't threaten to sue, but gets a payoff of -10 if they do threaten to sue

c. (3 points) Find all Pure Strategy Nash Equilibria (no need to find mixed strategy NE). Show your work.

NE:
Part II. Ann and Bob each have four cards, numbered 1, 2, 3, and 4. They each choose one of their cards and show them simultaneously. Whoever's card has the higher number wins the difference between the two cards in dollars. The loser pays the winner the dollars. For example, if Ann shows a 4 and Bob shows a 1, Bob pays Ann 3 dollars. If there is a draw, then no one pays anyone.

a. Represent this as a strategic form game. (3 points)

b. Make a prediction in this game using (i) iterative elimination of strongly or weakly dominated strategies (show the order of elimination) and (ii) pure strategy Nash equilibrium. (1.5 points each)
c. Now say that the game changes. Now if Ann and Bob’s cards are both even or both odd, a draw is declared and no one pays anyone anything. Otherwise the game is the same as before. For example, if Ann shows a 2 and Bob shows a 4, a draw is declared. If Ann shows a 4 and Bob shows a 1, Bob pays Ann 3 dollars. Represent this as a strategic form game. Can we use iterative elimination of strongly or weakly dominated strategies to make a prediction in this game? If so, show the order of elimination. Are the pure strategy Nash equilibria the same as before? (3 points)
d. Now start with the original game in part a. above and make the following change. Now if both Ann and Bob show a 4, then they both lose three dollars. Otherwise, the game is the same as in part a. Represent this as a strategic form game. Eliminate strategies by using the method of iterative elimination of strongly or weakly dominated strategies (show the order of elimination). In the remaining game, find all (pure strategy and mixed strategy) Nash equilibria. (3 pts)
Part III. Consider the following game.
Payoffs are written as (Player 1, Player 2, Player 3).

a. List all strategies for each player (4 points).
   Player 1’s strategies:
   Player 2’s strategies:
   Player 3’s strategies:

b. Represent this game as a strategic form game and find all Pure Strategy Nash Equilibria (4 points).
c. Find all Subgame Perfect Nash Equilibria (if there is more than one, write down all of them). (4 points)
Part IV. Consider the following game.

<table>
<thead>
<tr>
<th>Player 1</th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>x, 2</td>
<td>0, 4</td>
</tr>
<tr>
<td>D</td>
<td>4, 0</td>
<td>x, -2</td>
</tr>
</tbody>
</table>

a. Assume that x = -2. Find all pure strategy Nash equilibria (PSNE) and mixed strategy Nash equilibria (MSNE). (4 points)
b. In general, the Nash equilibria of the game of course depend on the value of $x$. Some possible values of $x$ are shown in the column below. Circle all the values of $x$ in the column below for which the game has no pure strategy Nash equilibria. Please explain your work. (4 points)

$x = -5$
$x = -4$
$x = -3$
$x = -2$
$x = -1$
$x = 0$
$x = 1$
$x = 2$
$x = 3$
$x = 4$
$x = 5$
c. Again, some possible values of x are shown in the column below. Now circle all the values of x in the column below for which the game has no mixed strategy Nash equilibria. Please explain your work. (4 points)

x = -5
x = -4
x = -3
x = -2
x = -1
x = 0
x = 1
x = 2
x = 3
x = 4
x = 5