Midterm exam  IEE 1149    July 2015

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, or digital devices of any kind are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

This exam has four parts. Each part is weighted equally (12 points each).

If you have a question, raise your hand and hold up the number of fingers which corresponds to the part you have questions about (if you have a question about question 2, hold up two fingers). If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
Part 1. Heejung and Minwoo are friends. Heejung lives near Yonsei while Minwoo lives in Kangnam. They are planning to go out on the town. Each person can either go to Hongdae, Sinchon, or Itaewon. Since they live in Seoul, “staying home” is not an option. Each person will take a cab. For Heejung, going to Hongdae costs 3000, going to Sinchon costs 2000, and going to Itaewon costs 6000. For Minwoo, going to Hongdae costs 10000, going to Sinchon costs 8000, and going to Itaewon costs 4000. Of course they enjoy each other’s company, and if they choose the same place, they both get a “bonus” of 5000. If they do not choose the same place, then neither person gets the bonus.

For example, if Heejung goes to Sinchon and Minwoo goes to Sinchon also, then Heejung gets a payoff of 3000 and Minwoo gets a payoff of -3000. If Heejung goes to Itaewon and Minwoo goes to Hongdae, then Heejung gets a payoff of -6000 and Minwoo gets a payoff of -10000.

They make their decisions simultaneously because their cell phones do not work.

a. Model this as a strategic form game. (3 points)

```
<table>
<thead>
<tr>
<th></th>
<th>Minwoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heejung</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>2000, 3000, 5000</td>
</tr>
<tr>
<td>S</td>
<td>-2000, -1000, 1000</td>
</tr>
<tr>
<td>I</td>
<td>-6000, -1000, -10000</td>
</tr>
</tbody>
</table>
```

b. Make a prediction in this game using iterative elimination of (strongly or weakly) dominated strategies. If you can eliminate (strongly or weakly) dominated strategies, eliminate as much as you can and show the order of elimination. If you cannot eliminate (strongly or weakly) dominated strategies, explain why not. (3 points)

1. For Minwoo, I s. dominates H
2. For Heejung, S s. dominates H
c. Find all pure strategy and mixed strategy Nash equilibria of this game. (Please note that an answer like \( p = 1/2, q = 1/3 \) is not sufficient to indicate a mixed Nash equilibrium—please write out the mixed Nash equilibrium completely.) (3 points)

We eliminated \( H \) for both players, so we have left:

\[
\begin{array}{c|cc}
& \text{Minwoo} & \text{Heejung} \\
\hline
S & -2,000 & -6,000 \\
I & -2,000 & -1,000 \\
\end{array}
\]

\[
\begin{align*}
3,000q + -2,000\left(1-q\right) &= -6,000q + -6,000\left(1-q\right) \\
3q - 2 + 2 &= -6q - 1 + 2 \\
5q - 2 &= -5 - 1 \\
10q &= 1 \\
q &= \frac{1}{10}
\end{align*}
\]

Mixed strategy NE: (\( M \) plays \( S \) with prob \( \frac{1}{10} \), \( M \) plays \( I \) with prob \( \frac{9}{10} \))

\[
\begin{array}{c|c|c|c}
& \text{Minwoo} & \text{Heejung} & \text{Pure strategy NEs:} \\
\hline
S & 5,5 & (5,5) \\
I & (1,1) & (1,1) \\
\end{array}
\]

d. Now say that Minwoo can get a ride with a friend who is going to Itaewon, and hence for Minwoo, the cost of going to Itaewon is now 1000. Minwoo’s costs for going to Hongdae or Sinchon are the same as before. Heejung’s costs are the same as before. Find all pure strategy and mixed strategy Nash equilibria of this game. (3 points)

\[
\begin{array}{c|c|c|c|c}
& \text{Minwoo} & \text{Heejung} & \text{Pure strategy NEs:} \\
\hline
S & 5,5 & (5,5) \\
I & (1,1) & (1,1) \\
\end{array}
\]

For Minwoo, \( I \) \( S \), dominates \( H \) and \( S \)

For Heejung, \( I \) \( S \), dominates \( H \) and \( S \)

Hence the only pure strategy NE is \((I, I)\)

There is no mixed strategy NE
Part 2. Consider the extensive form game below.

a. Find all subgame perfect Nash equilibria of this game. If there is more than one, you can just write down each one, or you can indicate each one by writing down arrows in a separate tree for each one. Note that payoffs are written as (Player 1, Player 2, Player 3). (3 points)

SPNE: C B, D E, H
Here is the game again.

b. Write down this game in strategic form and find all pure strategy Nash equilibria of this game. (3 points)

\[
\begin{array}{cccc}
& C & & \\
A & 2,0,6 & 2,0,6 & 7,1,7 & 7,1,7 \\
B & 8,9,4 & 7,3,2 & 8,9,4 & 7,3,2 \\
& F & & \\
& E & & \\
& D & & \\
& E & & \\
& F & & \\
\end{array}
\]

\[\text{NE: } (B, E, H) \]
\[\text{NE: } (B, D, E, H) \]
\[\text{NE: } (A, D, F, H) \]
Now consider this game.

Now consider this game.

Find all subgame perfect Nash equilibria of this game. If there is more than one, you can just write down each one, or you can indicate each one by writing down arrows in a separate tree for each one. (3 points)

(b, P, E, H)

(A, P, E, G)

(A, P, F, G)

(b, P, F, G)
Here is the game again.

d. Write down this game in strategic form and find all pure strategy Nash equilibria of this game. (3 points).

Pure strategy ME:

(A, D, C)  (B, E, H)
(A, D, C)  (B, E, H)
(B, C, E, G) (A, D, H)
(B, C, E, G) (A, D, H)
(B, C, F, G) (B, D, H)
Part 3. Persons 1, 2, and 3 are playing a word game. Person 1 chooses the first letter to be either “B” or “C.” Person 2 chooses the second letter to be either “A” or “E.” Person 3 chooses the third letter to be either “G” or “T.” When they choose their letters, they might or might not form a word in the English language. For example, if person 1 chooses C, person 2 chooses A, and person 3 chooses T, then they spell CAT, which is a word. If person 1 chooses C, person 2 chooses E, and person 3 chooses T, then they spell CET, which is not a word. Note that CAG, CEG, and CET are not words.

Person 1 is the “early” player: person 1 wants a word to be spelled, and he wants the word to be as early in the dictionary as possible. For example, person 1 prefers BAG over BEG. Person 3, however, is the “late” player: she wants a word to be spelled and for the word to be as late in the dictionary as possible. For example, person 3 prefers BAT over BAG. For both the “early” and the “late” player, not spelling a word is the worst possible outcome; any outcome which is not a word is equally bad.

Person 2 is a “spoiler” and does not want a word to be spelled. Person 2 does not care about where the word falls in the dictionary. If a word is spelled, that is a failure for person 2 and it does not matter what the word is; all are equally bad.

a. Model this as a strategic form game. Use the numbers 0, 1, 2, 3, 4, 5 for person 1’s payoffs and person 3’s payoffs. Use the numbers 0 and 5 for person 2’s payoffs. (2 points)

b. Find all pure strategy Nash equilibria of this game. (2 points)
c. Now say that person 1 is the “late” player, person 2 is the “early” player, and person 3 is the “spoiler.” Model this as a strategic form game. Use the numbers 0, 1, 2, 3, 4, 5 for person 1’s payoffs and person 2’s payoffs. Use the numbers 0 and 5 for person 3’s payoffs. (2 points)

\[
\begin{array}{cc|cc}
 & A & E \\
B & (1, 5, 0) & 3, 3, 0 \\
C & 0, 0, 5 & 0, 0, 5 \\
\end{array}
\quad \quad
\begin{array}{cc|cc}
 & A & E \\
B & 2, 4, 0 & 4, 2, 0 \\
C & 5, 1, 0 & 0, 0, 5 \\
\end{array}
\]

d. Find all pure strategy Nash equilibria of this game. (2 points)

\[
(C, B, A, C)
\]

e. Now say that you are the party host and can assign the “early,” “late,” and “spoiler” roles to whomever you wish. For example, you can make person 1 the “spoiler,” person 2 the “late” player, and person 3 the “early” player. Note that you must have exactly one “early” player, one “late” player, and one “spoiler”—you cannot have two “spoilers” and one “early” player, for example.

Say that you like person 1 and want person 1 to “win,” that is, get her highest possible payoff. Is it possible for you to assign the “early,” “late,” and “spoiler” roles so that person 1 wins? If so, write down which person gets which role and show that there exists a pure strategy Nash equilibrium in which person 1 gets her highest possible payoff. If not, explain why not. (2 points)

1 spoiler
2 early
3 late

1 doesn’t want to deviate to B
2 doesn’t gain by deviating to A
3 “other correct answers are possible”
f. Again, say you are the party host and can assign the “early,” “late,” and “spoiler” roles to whomever you wish. Say that now you like person 2 and want person 2 to “win,” that is, get her highest possible payoff. Is it possible for you to assign the “early,” “late,” and “spoiler” roles so that person 2 wins? If so, write down which person gets which role and show that there exists a pure strategy Nash equilibrium in which person 2 gets her highest possible payoff. If not, explain why not. (1 point)

\[ \text{In part c, person } 1 \text{ is late, } 2 \text{ early, and } 3 \text{ spoiler} \]

and \((B, A, C)\) is a NE in which person 2 gets 5

\[
\text{[other correct answers are possible]}\]

g. Again, say you are the party host and can assign the “early,” “late,” and “spoiler” roles to whomever you wish. Say that now you like person 3 and want person 3 to “win,” that is, get his highest possible payoff. Is it possible for you to assign the “early,” “late,” and “spoiler” roles so that person 3 wins? If so, write down which person gets which role and show that there exists a pure strategy Nash equilibrium in which person 3 gets his highest possible payoff. If not, explain why not. (1 point)

\[
\begin{align*}
\text{If} & \\
1 \text{ spoiler} & \\
2 \text{ early} & \\
3 \text{ late} & \\
\text{then } (C, A, I) \text{ is a NE} & \\
1 \text{ does not gain by deviating to } B & \\
2 \text{ loses by deviating to } E & \\
3 & \quad \text{“} & \quad \text{“}
\end{align*}
\]

\[
\text{[other correct answers are possible]}\]
Part 4. The Eurozone and Greece are currently negotiating the terms of a financial bailout for Greece. The Eurozone first decides whether to offer harsh terms or soft terms, where harsh terms are less favorable for Greece and soft terms are more favorable to Greece. Once the Eurozone makes its offer, Greece can then decide whether to accept or refuse, as in the game below.

The worst thing for the Eurozone is to offer harsh terms and then have Greece refuse, because then the Eurozone is embarrassed in the eyes of the world (it looks excessively harsh). If there is a deal, then the Eurozone prefers harsh terms over soft terms. The best thing for Greece to is accept soft terms. The second best thing for Greece is to accept harsh terms. If Greece refuses harsh terms, it gets some sympathy from the world community.

a. Fill in the payoffs in blanks in the tree above. For the Eurozone’s payoffs, use the numbers -5, 5, and 10. For Greece’s payoffs, use the numbers 3, 5, and 10. Payoffs are (Eurozone, Greece). (2 points)

b. Find all subgame perfect equilibria of this game. (2 points)

c. Find all pure strategy Nash equilibria of this game. (2 points)

\[
\begin{array}{c|c|c}
    & \text{accept} & \text{refuse} \\
\hline
\text{harsh} & (10, 5) & (-5, 3) \\
\text{soft} & (5, 10) & (0, 0) \\
\end{array}
\]

Pure strategy NE: (H, \text{accept}) (H, a) (S, a)
Say that we have this model of the situation.

Here the variable $x$ indicates the Eurozone’s embarrassment from “forcing” Greece to accept harsh terms. The variable $y$ indicates Greek pride—how much the Greek government will be rewarded by voters if it refuses harsh terms from the Eurozone.

d. Say that $x = 0$ and $y = 0$. Find all subgame perfect Nash equilibria of the game. (3 points)

$$(H, \alpha)$$
e. Say that $x$ and $y$ depend on the international political climate. Below are some values of $x$ and $y$. Circle all which give you a subgame perfect Nash equilibrium in which the Eurozone offers soft terms and Greece accepts.

For example, when $x = 2$ and $y = 4$, if there is a subgame perfect Nash equilibrium in which the Eurozone offers soft terms and Greece accepts, then circle $x=2, y=4$. Please explain your work. (3 points)