Answers to
Quiz 1       PS 172       February 2014

Name:

UID number:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, or any electronic or computational devices are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has two parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
1. Say that persons 1 and 2 are thinking about whether to bring an umbrella (b) or not (n) to work. Today’s weather calls for either sun, rain, or a hailstorm—each of these three possibilities is equally likely. Payoffs are given as follows.

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**Sun**  **Rain**  **Hailstorm**

Note that a person always gets -6 for bringing an umbrella. If it is sunny, and neither person brings an umbrella, then both get 0 (nothing happens because it is sunny). If it is rainy, and neither person brings an umbrella, then both get wet and get -18. If it is hailing, then each person needs her own umbrella: even if the other person brings an umbrella, if you don’t bring yours, you get -24.

a. Say that neither person knows anything about the weather today. Represent this as a strategic form game and find all Nash equilibria. (2 points)

\[ \text{NE: } (b b b, b b b) \]

b. Now say that both people know whether it is sunny or not, but do not know whether it is raining or hailing. Represent this as a strategic form game and find all Nash equilibria. (3 points)

\[ \text{NE: } (n bb, n bb) \]
Here are the payoffs again for your convenience.

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<td>-18, -18</td>
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<td>n</td>
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*Sun*  
*Rain*  
*Hailstorm*

c. Now say that both people know whether it is hailing or not, but do not know whether it is sunny or raining. Represent this as a strategic form game and find all Nash equilibria. (3 points)

d. Now say that person 1 does not know anything about the weather. Person 2 knows whether it is hailing or not, but does not know if it is sunny or raining. Represent this as a strategic form game and find all Nash equilibria. (3 points)

e. Of the four scenarios above (parts a, b, c, and d), which is best for person 2? (1 point)

Part d. is best for 2 because she gets -2 for sure (the only NE).
2. Say that two people are deciding how many hours to wait in line to get tickets for this summer’s EDM festival. Person 1 chooses to wait \( a_1 \) hours and person 2 chooses to wait \( a_2 \) hours. The headliner will either be Skrillex or Tiësto. The probability that Skrillex is the headliner is 1/2 and the probability that Tiësto is the headliner is 1/2.

Person 1 likes Skrillex but is a huge Tiësto fan. If Skrillex is the headliner, person 1’s utility function is \( u_1(s, a_1, a_2) = (12 + a_2) a_1 - (a_1)^2 \). If Tiësto is the headliner, person 1’s utility function is \( u_1(t, a_1, a_2) = (24 + a_2) a_1 - (a_1)^2 \).

Person 2 likes Skrillex and Tiësto equally. Regardless of who is the headliner, person 2’s utility function is \( u_2(a_1, a_2) = (12 + a_1) a_2 - (a_2)^2 \).

a. Say that neither person knows who the headliner will be. Model this as a game and find Nash equilibria. (3 points)

\[
\begin{align*}
\mathbb{E}u_1 &= \frac{1}{2} \left( (12+e_2) a_1 - (a_1)^2 \right) + \frac{1}{2} \left( (24+e_2) a_1 - (a_1)^2 \right) \\
\frac{d\mathbb{E}u_1}{da_1} &= \frac{1}{2} \left( 12 + a_2 - 2a_1 \right) + \frac{1}{2} \left( 24 + a_2 - 2a_1 \right) = 0 \\
12 + a_2 - 2a_1 + 24 + a_2 - 2a_1 &= 0 \\
24 + 2a_2 &= 4a_1 \\
9 + \frac{a_1}{2} &= a_1 \\
2a_2 &= 9 - \frac{a_1}{2} \\
\end{align*}
\]

\[
\begin{align*}
\mathbb{E}u_2 &= \frac{1}{2} \left( (12+e_1) a_2 - (a_2)^2 \right) + \frac{1}{2} \left( (12+e_1) a_2 - (a_2)^2 \right) \\
\frac{d\mathbb{E}u_2}{da_2} &= \frac{1}{2} \left( 12 + a_1 - 2a_2 \right) + \frac{1}{2} \left( 12 + a_1 - 2a_2 \right) = 0 \\
12 + a_1 - 2a_2 + 12 + a_1 - 2a_2 &= 0 \\
24 + 2a_1 - 4a_2 &= 0 \\
6 + \frac{a_1}{2} &= a_2 \\
\end{align*}
\]

\[
\begin{align*}
\text{Solution:} & \quad 9 + \frac{1}{2} \left( 6 + \frac{a_1}{2} \right) = a_1 \\
16 + 6 + \frac{a_1}{2} &= 2a_1 \\
2a_1 &= 3a_1 - \frac{a_1}{2} \implies a_1 = \frac{4a_1}{3} = 16, \quad a_2 = 6 + \frac{16}{2} = 14. \\
\end{align*}
\]

So, NE is \((16, 16), (14, 14)\).
b. Now say that both people know who the headliner will be. Model this as a game and find Nash equilibria. (2 points)

\[ EU_1 = \frac{1}{2} \left( (12 + a_2^s) a_1^s - (a_1^s)^2 \right) + \frac{1}{2} \left( (2q + a_2^t) a_1^t - (a_1^t)^2 \right) \]

\[ \frac{dE}{da_1^s} = \frac{1}{2} \left( 12 + a_2^s - 2a_1^s \right) = 0 \]
\[ 12 + a_2^s - 2a_1^s = 0 \]
\[ a_1^s = \frac{12 + a_2^s}{2} \]

\[ \frac{dE}{da_1^t} = \frac{1}{2} \left( 2q + a_2^t - 2a_1^t \right) = 0 \]
\[ 2q + a_2^t - 2a_1^t = 0 \]
\[ a_1^t = \frac{2q + a_2^t}{2} \]

\[ EU_2 = \frac{1}{2} \left( (12 + a_1^s) a_2^s - (a_2^s)^2 \right) + \frac{1}{2} \left( (12 + a_1^t) a_2^t - (a_2^t)^2 \right) \]

\[ \frac{dE}{da_2^s} = \frac{1}{2} \left( 12 + a_1^s - 2a_2^s \right) = 0 \]
\[ 12 + a_1^s - 2a_2^s = 0 \]
\[ a_2^s = \frac{12 + a_1^s}{2} \]

\[ \frac{dE}{da_2^t} = \frac{1}{2} \left( 12 + a_1^t - 2a_2^t \right) = 0 \]
\[ 12 + a_1^t - 2a_2^t = 0 \]
\[ a_2^t = \frac{12 + a_1^t}{2} \]

Solving:
\[ 6 + \frac{1}{2} \left( 6 + \frac{a_1^s}{2} \right) = a_1^s \]
\[ 12 + 6 + \frac{a_1^s}{2} = 2a_1^s \]
\[ x = 2 \frac{a_1^s}{2} \]
\[ a_1^s = \frac{12}{2} \]
\[ a_1^t = \frac{12 + \frac{20}{2}}{2} = 16 \]

So, NE is \((12, 20), (12, 16)\)
c. Now say that person 1 knows who the headliner will be but person 2 does not. Model this as a game and find Nash equilibria. (3 points)

\( P_1 \) \( a_s^1 \) \( a_s^2 \)

\( E_u_1 = \frac{1}{2} \left( (12 + a_s) a_s^1 - (a_s^1)^2 \right) + \frac{1}{2} \left( (12 + a_s) a_s^2 - (a_s^2)^2 \right) \)

\( \frac{dE_u_1}{da_s^1} = \frac{1}{2} \left( 12 + a_s - 2a_s^1 \right) = 0 \)

\( 12 + a_s - 2a_s^1 = 0 \)

\( 12 + a_s = 2a_s^1 \)

\( a_s^1 = 6 + \frac{a_s}{2} \)

\( \frac{dE_u_1}{da_s^2} = \frac{1}{2} \left( 24 + a_s - 2a_s^2 \right) = 0 \)

\( 24 + a_s - 2a_s^2 = 0 \)

\( a_s = 24 + 2a_s^2 \)

\( a_s^2 = \frac{24 + a_s}{2} \)

\( a_s^2 = \frac{24 + 24}{2} = 14 \)

\( a_s^1 = 6 + \frac{14}{2} = 13 \)

\( a_s^2 = 14 + \frac{14}{2} = 19 \)

\( \text{NE is} \) \( (13, 14), (14, 14) \)
d. Now say that person 2 knows who the headliner will be but person 1 does not. Model this as a game and find Nash equilibria. (3 points)

\[ E_{u1} = \frac{1}{2} (12 + a_2) a_1 - (c_1)^2 + \frac{1}{2} (2c + a_2^+) a_1^2 - (c_1)^2 \]

\[ \frac{dE_{u1}}{da_1} = \frac{1}{2} (12 + a_1 - 2a_2^+) - \frac{1}{2} (2c + a_2^+) = 0 \]

\[ 12 + a_1 - 2a_2^+ = 0 \]

\[ 12 + a_1 = 2a_2^+ \]

\[ 6 + a_1 = a_2^+ \]

\[ E_{u2} = \frac{1}{2} (12 + a_1) a_2^+ - (c_1)^2 + \frac{1}{2} (12 + a_1) a_2^+ - (c_2)^2 \]

\[ \frac{dE_{u2}}{da_2^+} = \frac{1}{2} (12 + a_1 - 2a_2^+) = 0 \]

\[ 12 + a_1 - 2a_2^+ = 0 \]

\[ 12 + a_1 = 2a_2^+ \]

\[ 6 + a_1 = a_2^+ \]

Solving

\[ 9 + \frac{1}{2} (6 + \frac{a_2^+}{2} + 6 + \frac{a_2^+}{2}) = a_1 \]

\[ 36 + 6 + \frac{a_2^+}{2} + 6 + \frac{a_2^+}{2} = 4a_1 \]

\[ 48 + a_2^+ = 4a_1 \]

\[ 4a_1 = 3a_1 \rightarrow a_1 = 16 \]

\[ a_2^+ = 6 + \frac{16}{2} = 14 \]

\[ a_2^+ = 6 + \frac{16}{2} = 14 \]

So NE is \((16, 16), (14, 14)\)
e. Which of these scenarios above (a., b., c., and d.) is best for person 1? (1 point)

\[ EU_1 = \frac{1}{2} \left( (12 + 3^2) a_1 - \left( a_1 \right)^2 \right) + \frac{1}{2} \left( (12 + 4^2) a_1 - \left( a_1 \right)^2 \right) \]

\[ EU_2 = \frac{1}{2} \left( (12 + k^2) a_2 - \left( a_2 \right)^2 \right) + \frac{1}{2} \left( (12 + k^2) a_2 - \left( a_2 \right)^2 \right) \]

**Part a:** \((16, 16), (14, 14)\)

\[ EU_1 = \frac{1}{2} \left( (12 + 14) 16 - \left( 16 \right)^2 \right) + \frac{1}{2} \left( (12 + 14) 16 - \left( 16 \right)^2 \right) \]

\[ = \frac{1}{2} \left( 24 \cdot 16 - 16 \cdot 16 \right) + \frac{1}{2} \left( 24 \cdot 16 - 16 \cdot 16 \right) \]

\[ = \frac{1}{2} \cdot 10 \cdot 16 + \frac{1}{2} \cdot 22 \cdot 16 \]

\[ = 5 \cdot 16 + 11 \cdot 16 = 16 \cdot 16 = 256 \]

\[ EU_2 = \frac{1}{2} \left( (12 + 16) 14 - \left( 14 \right)^2 \right) + \frac{1}{2} \left( (12 + 16) 14 - \left( 14 \right)^2 \right) \]

\[ = \frac{1}{2} \left( 28 \cdot 14 - 14 \cdot 14 \right) + \frac{1}{2} \left( 28 \cdot 14 - 14 \cdot 14 \right) \]

\[ = \frac{1}{2} \left( 194 \cdot 14 \right) = \frac{1}{2} \cdot 14 = 96 \]

**Part b:** \((19, 20), (13, 16)\)

\[ EU_1 = \frac{1}{2} \left( (12 + 12) 20 - \left( 20 \right)^2 \right) + \frac{1}{2} \left( (12 + 12) 20 - \left( 20 \right)^2 \right) \]

\[ = \frac{1}{2} \left( 24 \cdot 20 - 20 \cdot 20 \right) + \frac{1}{2} \left( 24 \cdot 20 - 20 \cdot 20 \right) \]

\[ = \frac{1}{2} \cdot 12 \cdot 20 + \frac{1}{2} \cdot 20 \cdot 20 = \frac{1}{2} \cdot 144 + \frac{1}{2} \cdot 400 = 72 + 200 = 272 \]

**Part c:** \((13, 14), (14, 14)\)

\[ EU_1 = \frac{1}{2} \left( (12 + 19) 13 - \left( 13 \right)^2 \right) + \frac{1}{2} \left( (12 + 19) 13 - \left( 13 \right)^2 \right) \]

\[ = \frac{1}{2} \left( 26 \cdot 13 - 13 \cdot 13 \right) + \frac{1}{2} \left( 26 \cdot 13 - 13 \cdot 13 \right) \]

\[ = \frac{1}{2} \cdot 169 + \frac{1}{2} \cdot 361 = \frac{1}{2} \cdot 530 = 265 \]

\[ EU_2 = \frac{1}{2} \left( (12 + 14) 14 - \left( 14 \right)^2 \right) + \frac{1}{2} \left( (12 + 14) 14 - \left( 14 \right)^2 \right) \]

\[ = \frac{1}{2} \left( 25 \cdot 14 - 14 \cdot 14 \right) + \frac{1}{2} \left( 25 \cdot 14 - 14 \cdot 14 \right) \]

\[ = \frac{1}{2} \cdot (31 \cdot 14) = \frac{1}{2} \cdot 14 = 7 \]

**Part d:** \((16, 16), (19, 19)\) same as part a.

So, person 1 (and person 2) like part b best (even we know everything).
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