Final exam PS 172 March 2012

Name:

This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, etc. are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has five parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
1. Say that a PTA council is composed of parents and teachers. The council approves a budget if at least one parent and one teacher vote for it. It does not matter how many total votes there are as long as at least one parent and one teacher vote for it.

a. Say that the council has three people: two parents (persons 1 and 2) and one teacher (person 4). So \( v(\{1, 4\}) = 1 \) (a parent and teacher can pass a budget) but \( v(\{1, 2\}) = 0 \) (two parents alone cannot pass a budget). We also have \( v(\{1, 2, 4\}) = 1 \) (as long as at least one parent and one teacher vote for it, the budget passes). Find the Shapley value of this game. (3 points)

b. Now say that another parent, person 3, is elected to the council. Now the council has four members: persons 1, 2, and 3 are parents and person 4 is a teacher. Like before, \( v(\{1, 4\}) = 1 \) for example, but \( v(\{1, 3\}) = 0 \) (two parents cannot pass a budget). Also, we still have \( v(\{1, 2, 3, 4\}) = 1 \). Find the Shapley value of this game. (3 points)
c. Now say that person 5, another teacher, is elected to the council. Now the PTA council has five members: persons 1, 2, and 3 are parents and persons 4 and 5 are teachers. Again, the council approves a budget if at least one parent and one teacher vote for it. Like before, $v(\{1, 4\}) = 1$ for example, but $v(\{1, 3\}) = 0$ (two parents cannot pass a budget) and $v(\{4, 5\}) = 0$ (two teachers cannot pass a budget). Also, we still have $v(\{1, 2, 3, 4, 5\}) = 1$. Find the Shapley value of this game. (3 points)

d. Finally, now say that the voting rule changes. Now it takes two parents and two teachers to approve a budget. Like before, persons 1, 2, and 3 are parents and persons 4 and 5 are teachers. So now $v(\{1, 4\}) = 0$ and $v(\{1, 2, 4, 5\}) = 1$. We also have $v(\{1, 2, 3, 4\}) = 0$ (three parents and one teacher cannot pass a budget) and $v(\{1, 2, 3, 4, 5\}) = 1$. Find the Shapley value of this game. (3 points)
2. Say that there are two firms which sell Lakers jerseys. Firm 1 produces quantity $q_1$ and Firm 2 produces quantity $q_2$. Given these quantities, the market price is $120 - (q_1 + q_2)$. Hence Firm 1’s utility function is $u_1(q_1, q_2) = (120 - (q_1 + q_2))q_1$ and Firm 2’s utility function is $u_2(q_1, q_2) = (120 - (q_1 + q_2))q_2$ (there are no production costs).

a. Find the Nash equilibrium of this game. (3 points)
b. Now say that there are three firms. Firm 1 produces quantity \( q_1 \), Firm 2 produces quantity \( q_2 \), and Firm 3 produces quantity \( q_3 \). Given these quantities, the market price is \( 120 - (q_1 + q_2 + q_3) \). Thus \( u_1(q_1, q_2, q_3) = (120 - (q_1 + q_2 + q_3))q_1 \), \( u_2(q_1, q_2, q_3) = (120 - (q_1 + q_2 + q_3))q_2 \), and \( u_3(q_1, q_2, q_3) = (120 - (q_1 + q_2 + q_3))q_3 \). Find the Nash equilibrium of this game. (3 points)
c. Remember that there are three firms. Now say that Jeremy Lin is rumored to be coming to the Lakers. If Lin becomes a Laker, demand will increase and the market price will be \(240 - (q_1 + q_2 + q_3)\) instead. If Lin remains with the Knicks, the market price remains at \(120 - (q_1 + q_2 + q_3)\). The probability that Lin is coming to the Lakers (state \(L\)) is \(1/2\) and the probability that he is staying with the Knicks (state \(K\)) is \(1/2\). However, Firm 1 has inside information on the trade and knows whether Lin is coming or not. Firm 2 and Firm 3 know nothing. Find the Bayesian Nash equilibrium of this game. (3 points)
d. Now both Firm 1 and Firm 2 have inside information on the trade and know whether Lin is coming or not. Firm 3 knows nothing. Find the Bayesian Nash equilibrium of this game. (3 points)
3. Say that UCLA and USC are bargaining over how to split television revenues from a crosstown dodgeball match. They can get $900,000 if they set up the match. However, if UCLA simply does an intramural dodgeball match by itself, it can get $300,000 and if USC sets up an intramural dodgeball match by itself, it can get $100,000. Say that UCLA’s discount factor is $\delta_{UCLA} = \frac{4}{5}$ and USC’s discount factor is $\delta_{USC} = \frac{3}{5}$.

a. Say that they bargain using alternating-offer bargaining. What is the result if UCLA makes the first offer? What is the result if USC makes the first offer? (6 points)
b. Now say that only two rounds of bargaining are possible. First UCLA makes an offer, then USC can accept or reject it. If USC rejects it, then it makes a final offer to UCLA, which UCLA can accept or reject. If UCLA rejects it, then bargaining is over and there is no crosstown match. What is the result of this bargaining procedure? (3 points)
c. Now say that only three rounds of bargaining are possible. First UCLA makes an offer, then USC can accept or reject it. If USC rejects it, then it makes an offer to UCLA, which UCLA can accept or reject. If UCLA rejects it, then it makes a final offer to USC. If USC rejects this offer, then bargaining is over and there is no crosstown match. What is the result of this bargaining procedure? (3 points)
4. Consider the two-person game below.

a. Find all Nash equilibria of this game. (6 points).
b. Find all Perfect Bayesian Nash equilibria of this game. I write it down several times so you don’t have to spend time writing the trees over and over again. (6 points)
5. Say that two people each decide whether to go out on the town \( t \) or stay home \( s \). If you stay home, you get a payoff of 0. If you go out on the town alone, then you are unhappy and get a payoff of \(-4\). If you both decide to go out, then the two of you get picked up by a mystery cab which can take the two of you to four possible places: karaoke \( k \), blues bar \( b \), disco \( d \), or cigar lounge \( c \). Each location is equally likely (each takes place with probability \( 1/4 \)). The payoffs from each location look like this:

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<tr>
<th></th>
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<th>( t )</th>
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</thead>
<tbody>
<tr>
<td>( t )</td>
<td>(-12, 28)</td>
<td>(-4, 0)</td>
<td>(8, -8)</td>
<td>(-4, 0)</td>
<td>(16, -8)</td>
<td>(-4, 0)</td>
<td>(-4, -4)</td>
<td>(-4, 0)</td>
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<tr>
<td>( s )</td>
<td>(0, -4)</td>
<td>(0, 0)</td>
<td>(0, -4)</td>
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<td>(0, -4)</td>
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**Karaoke**  **Blues**  **Disco**  **Cigar**

a. Say that neither person knows where the mystery cab will go to before deciding whether to go out or not. Represent this as a game and find all Nash equilibria. (4 points)
b. Now say that person 1 doesn’t know where the mystery cab will go to, but person 2 knows ahead of time whether the mystery cab will go to karaoke or not (person 2 can’t tell whether the mystery cab will go to the blues bar, disco, or cigar lounge, but she can tell whether it will go to karaoke or not). Represent this as a game and find all Nash equilibria. Here is the game again for your reference. (4 points)

<table>
<thead>
<tr>
<th></th>
<th>Karaoke</th>
<th>Blues</th>
<th>Disco</th>
<th>Cigar</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>12,28</td>
<td>8,8</td>
<td>16,8</td>
<td>-4,4</td>
</tr>
<tr>
<td>( s )</td>
<td>-4,0</td>
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\[
\begin{array}{cccc}
  t & s & t & s & t & s & t & s \\
  -12 & 28 & -4 & 0 & 8 & -8 & -4 & 0 \\
  0 & -4 & 0 & 0 & 0 & -4 & 0 & 0 \\
  Karaoke & Blues & Disco & Cigar \\
\end{array}
\]
c. Now consider all possible knowledge partitions for person 1 and for person 2. Draw the partitions for person 1 and for person 2 which yield a Bayesian Nash equilibrium which is best for person 1. Draw the partitions for person 1 and for person 2 which yield a Bayesian Nash equilibrium which is best for person 2. Here is the game again for your reference. (4 points)

<table>
<thead>
<tr>
<th></th>
<th>t</th>
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<tr>
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<td>t</td>
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<td>s</td>
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<td>s</td>
<td>0, -4</td>
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</tr>
</tbody>
</table>

Karaoke | Blues | Disco | Cigar