1. Say that persons 1 and 2 are thinking about watching the Lakers-Knicks game tonight. They are both Jeremy Lin fans but it is 50-50 whether Lin will be in the game because he is suffering from a sore ankle. Person 1 does not have a Tivo machine but person 2 does. So person 1 decides whether to watch the game live w or not n, and person 2 decides whether to watch the game live w or Tivo the game t. Payoffs are given as follows.

\[
\begin{array}{c|cc}
 & t & \\
 w & 16,16 & 20,10 \\
n & 0,16 & 0,10 \\
\end{array}
\]

Lin in the game

\[
\begin{array}{c|cc}
 & t & \\
 w & 8,4 & 8,12 \\
n & 0,4 & 12,12 \\
\end{array}
\]

Lin not in the game

Note that if Lin is not in the game, the best thing for person 1 is if person 2 Tivos it so he can see it later at person 2's house; if person 2 does not Tivo it, then person 1 wants to watch the game. If Lin is in the game, person 1 wants to watch the game and also for person 2 to Tivo it so that he can go to person 2's house later to relive the experience.

a. Say that neither person knows whether Lin will be in the game or not. Represent this as a strategic form game and find all Nash equilibria.

\[
\begin{array}{c|c|c|c|c|c}
 & w & w & & & \\
 w & 12,10 & 14,11 & & & \\
n & 0,10 & 6,11 & & & \\
\end{array}
\]

\[
NE = \{(ww), (tt)\}
\]
Here are the payoffs again for convenience.

<table>
<thead>
<tr>
<th></th>
<th>w</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>16,16</td>
<td>20,10</td>
</tr>
<tr>
<td>n</td>
<td>0,16</td>
<td>0,10</td>
</tr>
</tbody>
</table>

Lin in the game

<table>
<thead>
<tr>
<th></th>
<th>w</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>8,4</td>
<td>8,12</td>
</tr>
<tr>
<td>n</td>
<td>0,4</td>
<td>12,12</td>
</tr>
</tbody>
</table>

Lin not in the game

b. Say that the New York Post reports ahead of time whether or not Lin will be in the game or not, and both people read the Post. Represent this as a strategic form game and find all Nash equilibria.

\[
\begin{array}{c|c|c|c|c}
\text{ww} & \text{wit} & \text{tww} & \text{tt} \\
\hline
12,10 & 14,7 & 14,11 & \ \\
\hline
8,10 & 14,14 & 10,7 & 16,11 \\
\hline
4,10 & 4,14 & 4,7 & 4,11 \\
\hline
0,10 & 6,14 & 0,7 & 6,11
\end{array}
\]

\[\text{WE: (ww, t)}\]

c. Say that only person 1 reads the Post and hence only person 1 knows whether Lin will be in the game. Represent this as a strategic form game and find all Nash equilibria.

\[
\begin{array}{c|c|c|c}
\text{ww} & \text{tt} \\
\hline
12,10 & 14,11 & \\
\hline
8,10 & 14,14 & 11 \\
\hline
4,10 & 4,11 & \\
\hline
0,10 & 6,11
\end{array}
\]

\[\text{WE: (w, t)}\]
Here are the payoffs again for convenience.

<table>
<thead>
<tr>
<th></th>
<th>( w )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>16, 16</td>
<td>20, 10</td>
</tr>
<tr>
<td>( n )</td>
<td>0, 16</td>
<td>0, 10</td>
</tr>
</tbody>
</table>

Lin in the game

<table>
<thead>
<tr>
<th></th>
<th>( w )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>8, 4</td>
<td>8, 12</td>
</tr>
<tr>
<td>( n )</td>
<td>0, 4</td>
<td>12, 12</td>
</tr>
</tbody>
</table>

Lin not in the game

d. Say that only person 2 reads the Post and hence only person 2 knows whether Lin will be in the game. Represent this as a strategic form game and find all Nash equilibria.

\[
\begin{array}{c}
\Phi_1 \quad \Phi_2 \\
\begin{array}{c}
(w, w) \\
(w, t) \\
(n, w) \\
(n, t)
\end{array}
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
& w & t & w & t \\
\hline
w & 12, 10 & 12, 14 & 14, 7 & 14, 11 \\
\hline
n & 0, 10 & 6, 14 & 0, 7 & 6, 11 \\
\end{array}
\]

\[
\text{NE: } (w, w), (w, t)
\]

e. Among a., b., c., and d., above, which is the best situation for person 1?

a. \(\text{NE: } (w, w), (n, n)\) with payoffs (14, 11)
b. \(\text{NE: } (w, n), (n, t)\) with payoffs (14, 14) \(\text{best for person 1}\)
c. \(\text{NE: } (w, w), (n, t)\) with payoffs (16, 11)
d. \(\text{NE: } (w, w), (n, t)\) with payoffs (12, 19)

Person 1 would like for person 2 to not know whether Lin will play because that way person 2 will always lose the game.
20. \( q_1 \) army, no army
   \( q_2 \) army, no army

   \[
   \begin{array}{c|cc}
   r & r & s & s \\
   \hline
   r & 10, 1 & -8, 0 & +5, 0^+
   \end{array}
   \]

   \( s \):

   \[
   \begin{array}{c|cc}
   r & r & s & s \\
   \hline
   r & 0, -8 & +5, 0^+ & -8, 0
   \end{array}
   \]

   NE: \((r, r), (s, s)\)

21. \( q_1 \) \( \bigcirc \) \( \bigcirc \) \( \bigcirc \) \( \bigcirc \)
   \( q_2 \) \( \bigcirc \) \( \bigcirc \) \( \bigcirc \) \( \bigcirc \)

   \[
   \begin{array}{c|cc|cc|cc|cc}
   r & r & r & s & s & s & s \\
   \hline
   r & 10, 1 & -8, 0 & -8, 0 & 10, 10 & -8, 0 & -8, 0 & -8, 0 \\
   s & 10, -8 & -8, 0 & 0, -8 & -8, 0 & -8, 0 & 0, -8 & 10, 10 \\
   \hline
   0, -8 & -8, 0 & 0, -8 & 0, -8 & 0, -8 & 0, -8 & 0, -8 \\
   \end{array}
   \]

   NE: \((s, s), (s, s)\)
2c. \( p_1 \) 0 0
\( p_2 \)

\[
\begin{array}{ccc|c}
R & S & \text{Payoff} \\
\hline
R & 4,0 & +1 & 0,0 \\
S & -4,1 & -1 & 0,0 \\
\hline
\end{array}
\]

NE: (S, S), (S, S)

---

2d. \( p_1 \) 0 0
\( p_2 \)

\[
\begin{array}{ccc|c}
R & S & \text{Payoff} \\
\hline
R & 10,1 & +1 & 0,0 \\
S & -1,0 & -1 & 0,0 \\
\hline
\end{array}
\]

NE: (S, S), (S, S)

---

2e. If 1 has partition 0 0, then the NE (from 1b above) are
\( (5,5) \), \( (5,5) \) or \( (0,5) \), \( (0,5) \)

If 1 has partition 0 0, then the NE (from 1b above) are
\( (0,5) \), \( (0,5) \) or \( (5,5) \), \( (5,5) \)

So 1 prefers 2 to not be informed, because he gets 10 or 0
(as opposed to 5 or 0).
2F.

If 2 has partition $00$, then

the NE (from 1c above) are

$\begin{bmatrix} 00 & (0) \end{bmatrix}$ or $\begin{bmatrix} 00 & 0 \end{bmatrix}$

If 2 has partition $00$, then the NE (from 1b above) are

$\begin{bmatrix} 00 & (0) \end{bmatrix}$ or $\begin{bmatrix} 00 & 0 \end{bmatrix}$

2 prefers to be informed because she gets 5 or 0

(instead of 1 1 or 0).
\[ E_{U_1}(q_1, q_2) = \frac{1}{2} ((48 - 2q_1 - 2q_2) q_1) + \frac{1}{2} ((60 - 2q_1 - 2q_2) q_2) \]

\[ E_{U_2}(q_1, q_2) = \frac{1}{2} ((48 - 2q_1 - 2q_2) q_2) + \frac{1}{2} ((60 - 2q_1 - 2q_2) q_1) \]

1's best response?

\[
\frac{dE_{U_1}}{dq_1} = \frac{1}{2} (48 - 2q_1 - 2q_2) + \frac{1}{2} (60 - 2q_1 - 2q_2) = 0
\]

\[ 48 - 2q_1 - 2q_2 + 60 - 2q_1 - 2q_2 = 0 \]

\[ 108 - 2q_1 - 2q_2 = 0 \]

So, \[ q_1 = 27 - \frac{q_2}{2} \]

2's best response?

\[
\frac{dE_{U_2}}{dq_2} = \frac{1}{2} (48 - 2q_1 - 2q_2) + \frac{1}{2} (60 - 2q_1 - 2q_2) = 0
\]

\[ 48 - 2q_1 - 2q_2 + 60 - 2q_1 - 2q_2 = 0 \]

\[ 108 - 2q_1 - 2q_2 = 0 \]

So, \[ q_2 = 27 - \frac{q_1}{2} \]

1's strategy:

\[ q_1 = 27 - \frac{1}{2} \left( 27 - \frac{q_1}{2} \right) \]

\[ 2q_1 = 54 - \left( 27 - \frac{q_1}{2} \right) = 27 + \frac{q_1}{2} \]

\[ \frac{3q_1}{2} = 27 = \frac{q_1}{2} \]

\[ q_1 = 18 \]

So, \[ q_2 = 27 - \frac{18}{2} = 18 \]

So, \( NE = (18, 18) \).
3b. \[ q_1 \quad \frac{1}{2} \quad q_1 \]
\[ q_2 \quad q_2 \quad q_2 \]

\[ EU_1 (\xi_1^l, \xi_1^h, \xi_2^l, \xi_2^h) \]
\[ = \frac{1}{2} \left( (48 - \xi_1^l - \xi_2^l) \xi_1^l \right) + \frac{1}{2} \left( (60 - \xi_1^h - \xi_2^h) \xi_2^h \right) \]

\[ EU_2 (\xi_1^l, \xi_1^h, \xi_2^l, \xi_2^h) \]
\[ = \frac{1}{2} \left( (48 - \xi_1^l - \xi_2^l) \xi_2^l \right) + \frac{1}{2} \left( (60 - \xi_1^h - \xi_2^h) \xi_2^h \right) \]

1's best response?
\[ \frac{dEU_1}{d\xi_1^l} = \frac{1}{2} \left( 48 - 2\xi_1^l - 2\xi_2^l \right) = 0 \]
\[ 48 - 2\xi_1^l = 2\xi_2^l \]
\[ \xi_1^l = 24 - \frac{2\xi_2^l}{2} \]

\[ EU_1 = \frac{1}{2} \left( 60 - \xi_1^h - \xi_2^h \right) = 0 \]
\[ 60 - \xi_1^h = 2\xi_2^h \]
\[ \xi_1^h = 30 - \frac{2\xi_2^h}{2} \]

2's best response?
\[ \frac{dEU_1}{d\xi_2^l} = \frac{1}{2} \left( 48 - 2\xi_1^l - 2\xi_2^l \right) = 0 \]
\[ 48 - 2\xi_1^l = 2\xi_2^l \]
\[ \xi_2^l = 24 - \frac{2\xi_1^l}{2} \]

\[ EU_2 = \frac{1}{2} \left( 60 - \xi_1^h - 2\xi_2^h \right) = 0 \]
\[ 60 - \xi_1^h = 2\xi_2^h \]
\[ \xi_2^h = 30 - \frac{2\xi_1^h}{2} \]

Solution:
\[ \xi_1^l = 24 - \frac{1}{2} (24 - \frac{2\xi_2^l}{2}) \]
\[ 2\xi_2^l = 48 - (24 - \frac{2\xi_2^l}{2}) \]
\[ 2\xi_2^l = 24 + \frac{2\xi_1^l}{2} \]
\[ \frac{3}{2} \xi_2^l = 24 \Rightarrow \xi_1^l = 16 \]
\[ \xi_2^l = 24 - \frac{16}{2} = 16 \]
\[ \xi_1^h = 30 \Rightarrow 2\xi_2^h = 20 \]
\[ \xi_2^h = 10 \Rightarrow 2\xi_1^h = 20 \]

So NE is \((16, 20), (13, 20)\)
\[ \mathbf{EU}_1 (\xi^l_1, \xi^h_1, \xi^l_2, \xi^h_2) = \frac{1}{2} \left( 48 - 2\xi^l_1 - 2\xi^l_2 \right) + \frac{1}{2} \left( 60 - 2\xi^h_1 - 2\xi^h_2 \right) \]
\[ \mathbf{EU}_2 (\xi^l_1, \xi^h_1, \xi^l_2, \xi^h_2) = \frac{1}{2} \left( 48 - 2\xi^h_1 - 2\xi^h_2 \right) + \frac{1}{2} \left( 60 - 2\xi^l_1 - 2\xi^l_2 \right) \]

1's best response:

\[ \frac{d\mathbf{EU}_1}{d\xi^l_1} = \frac{1}{2} \left( 48 - 2\xi^l_1 - 2\xi^l_2 \right) = 0 \]
\[ 48 - 2\xi^l_1 - 2\xi^l_2 = 0 \]
\[ \xi^l_1 = 24 - \frac{2\xi^l_2}{2} \]

\[ \frac{d\mathbf{EU}_1}{d\xi^h_1} = \frac{1}{2} \left( 60 - 2\xi^h_1 - 2\xi^h_2 \right) = 0 \]
\[ 60 - 2\xi^h_1 - 2\xi^h_2 = 0 \]
\[ \xi^h_1 = 30 - \frac{2\xi^h_2}{2} \]

2's best response:

\[ \frac{d\mathbf{EU}_2}{d\xi^l_2} = \frac{1}{2} \left( 48 - 2\xi^l_1 - 2\xi^l_2 \right) + \frac{1}{2} \left( 60 - 2\xi^h_1 - 2\xi^h_2 \right) = 0 \]
\[ 48 - 2\xi^l_1 - 2\xi^l_2 + 60 - 2\xi^h_1 - 2\xi^h_2 = 0 \]
\[ 108 - 2\xi^l_1 - 2\xi^h_1 = 4 \xi^l_2 \]
\[ 9\xi^l_2 = 24 - \frac{1}{4} (2\xi^l_1 + 2\xi^h_1) \]

Solving:
\[ \xi^l_2 = 27 - \frac{1}{4} \left( 24 - \frac{18}{2} + 30 - \frac{20}{2} \right) \]
\[ 4\xi^l_2 = 108 - \left( 54 - \xi^l_2 \right) \]
\[ 4\xi^l_2 = 54 + \xi^l_2 \]
\[ 3\xi^l_2 = 54 \quad \Rightarrow \quad \xi^l_2 = 18 \]
\[ \xi^h_1 = 30 - \frac{18}{2} = 21 \]
\[ \xi^l_1 = 24 - \frac{18}{2} = 15 \]

So, NE is \((15, 21, 18, 18)\)
Part c.

From part c,

\[ EUI = \frac{1}{2} \left( (48 - 15 - 18) 15 \right) + \frac{1}{2} \left( (60 - 21 - 18) 21 \right) \]

\[ = \frac{1}{2} (15) + \frac{1}{2} (21) \]

\[ = \frac{1}{2} (225) + \frac{1}{2} (41) = \frac{1}{2} (666) = 333 \]

Part b.

From part b,

\[ EUI = \frac{1}{2} \left( (48 - 16 - 16) 16 \right) + \frac{1}{2} \left( (60 - 20 - 20) 20 \right) \]

\[ = \frac{1}{2} (16) + \frac{1}{2} (20) \]

\[ = \frac{1}{2} (256) + \frac{1}{2} (400) = \frac{1}{2} (656) = 328 \]

So firm 1 prefers the situation in part c (in which 2 is ignorant), because 333 > 328.

Part a.

From part a,

\[ EU_1 = \frac{1}{2} \left( (48 - 15 - 18) 15 \right) + \frac{1}{2} \left( (60 - 12 - 18) 18 \right) \]

\[ = \frac{1}{2} (15) + \frac{1}{2} (24) \]

\[ = \frac{1}{2} (216) + \frac{1}{2} (432) = \frac{1}{2} (648) = 324 \]

Part c.

\[ EU_2 = \frac{1}{2} \left( (48 - 15 - 18) 18 \right) + \frac{1}{2} \left( (60 - 21 - 18) 21 \right) \]

\[ = \frac{1}{2} (15) + \frac{1}{2} (21) \]

\[ = \frac{1}{2} (270) + \frac{1}{2} (378) = \frac{1}{2} (648) = 324 \]

Firm 2 doesn't care whether 1 knows or not, because 324 = 324.
4. Say that person 1 chooses a location $a_1$ and person 2 chooses a location $a_2$ on the beach, where $a_1$ and $a_2$ are both in the interval $[0, 1]$. The pier is at position $t = 0$ and the lifeguard station is at position $t = 1$. Person 1 likes to be close to person 2 but also the pier; hence her utility function is given by $u_1(a_1, a_2) = -(a_1 - a_2)^2 - (a_1 - 0)^2$. Person 2 likes to be close to person 1 and the lifeguard station. Hence person 2's utility function is $u_2(a_1, a_2, t) = -(a_2 - a_1)^2 - (a_2 - 1)^2$.

a. Model this as a game and find Nash equilibria.

$$u_1(a_1, a_2) = -(a_1 - a_2)^2 - a_1^2 = -a_1^2 + 2a_1a_2 - a_2^2$$

$$u_2(a_2, a_1) = -(a_2 - a_1)^2 - (a_2 - 1)^2 = -a_2^2 + 2a_2a_1 - a_1^2 - a_1^2 + 2a_1 - 1$$

1's best response:

$$\frac{du_1}{da_1} = -2a_1 + 2a_2 - 2a_1 = 0$$

$$4a_1 = 2a_2$$

$$a_1 = \frac{a_2}{2}$$

2's best response:

$$\frac{du_2}{da_2} = -2a_2 + 2a_1 - 2a_2 + 2 = 0$$

$$4a_2 = 2a_1 + 2$$

$$a_2 = \frac{a_1 + 1}{2}$$

NE:

$$a_1 = \frac{1}{2} \left( \frac{a_1 + 1}{2} \right)$$

$$2a_1 = \frac{a_1}{2} + \frac{1}{2}$$

$$\frac{3a_1}{2} = \frac{1}{2}$$

$$a_1 = \frac{1}{3}$$

$$a_2 = \frac{\frac{1}{3} + 1}{2} = \frac{2}{3}$$

\[ \begin{array}{c|c|c|c}
0 & \frac{1}{3} & \frac{2}{3} & 1 \\
\hline
\frac{1}{3} & a_1 & a_2 & \frac{2}{3} \\
\end{array} \]
b. Now say that the pier is selling hot dogs. If person 1 has a hot dog craving, his utility function is given by \( u_1(a_1, a_2, \text{craving}) = -(a_1 - a_2)^2 - 2(a_1 - 0)^2 \). If person 1 does not have a hot dog craving, his utility function is given by \( u_1(a_1, a_2, \text{not}) = -(a_1 - a_2)^2 - (a_1 - 0)^2 \) as before. It is equally likely whether person 1 has a craving or not. Say that person 1 knows whether he has a craving but person 2 does not know. Person 2’s utility function is the same as before. Model this as a game and find Nash equilibria.

\[
\mathcal{S} = \{ c, n \} \\
\mathcal{N} = \{ 1, 2 \}
\]

\( \mathcal{N}_1 \): strategy is \( \circ \circ \)

\( \mathcal{N}_2 \): strategy is \( a_2, a_2 \)

\[
\bar{\mathbb{E}}u_1(a_1^c, a_1^n, a_2) = \frac{1}{2} \left( -(a_1^c - a_2)^2 - 2(a_1^c)^2 \right) + \frac{1}{2} \left( -(a_1^n - a_2)^2 - (a_1^n)^2 \right)
\]

\[
= \frac{1}{2} \left( -(a_1^c)^2 + 2a_1^c a_2 - (a_2)^2 - 2(a_1^c)^2 \right)
\]

\[
+ \frac{1}{2} \left( -(a_1^n)^2 + 2a_1^n a_2 - (a_2)^2 - (a_1^n)^2 \right)
\]

\[
\frac{d\mathbb{E}u_1}{da_1^c} = \frac{1}{2} \left( -2a_1^c + 2a_2 - 4a_1^c \right) = 0
\]

\( a_1^c = \frac{a_2}{3} \)

\[
\frac{d\mathbb{E}u_1}{da_1^n} = \frac{1}{2} \left( -2a_1^n + 2a_2 - 2a_1^n \right) = 0
\]

\( 4a_1^n = 2a_2 \)

\( a_1^n = \frac{a_2}{2} \)

\[
\bar{\mathbb{E}}u_2(a_1^c, a_1^n, a_2) = \frac{1}{2} \left( -(a_1^c - a_2)^2 - (a_1^c - 1)^2 \right) + \frac{1}{2} \left( -(a_1^n - a_2)^2 - (a_1^n - 1)^2 \right)
\]

\[
= \frac{1}{2} \left( -a_1^c + 2a_2 a_1^c - (a_1^c)^2 - (a_1^c - 1)^2 \right)
\]

\[
+ \frac{1}{2} \left( -a_1^n + 2a_2 a_1^n - (a_1^n)^2 - (a_1^n - 1)^2 \right)
\]

\[
\frac{d\mathbb{E}u_2}{da_2} = \frac{1}{2} \left( -2a_2 + 2a_1^c - 2a_1^n + 2 \right) + \frac{1}{2} \left( -2a_2 + 2a_1^c - 2a_1^n + 2 \right) = 0.
\]
\[-2a_2 + 2a_1 = 2a_2 + 2 + -2a_2 + 2a_1 = 0\]

\[8a_2 = 2a_1 = 2a_1 \cdot \frac{1}{4} + \frac{1}{2}\]

\[a_2 = \frac{a_1 + a_1}{4} + \frac{1}{2}\]

\[a_2 = \frac{1}{4} \left( \frac{a_1}{3} + \frac{a_1}{2} \right) + \frac{1}{2}\]

\[c_2 \left( 1 - \frac{1}{12} - \frac{1}{4} \right) = \frac{1}{2}\]

\[c_2 \left( \frac{24 - 2 - 3}{24} \right) = \frac{1}{2}\]

\[\Rightarrow c_2 \left( \frac{19}{24} \right) = \frac{1}{2}\]

\[a_2 = \frac{12}{19}\]

\[a_4 = \frac{4}{19}\]

\[a_4 = \frac{6}{19}\]

\[NE: (\frac{4}{19}, \frac{6}{19}, \frac{12}{19}, \frac{12}{19})\]

\[\frac{a_1 \cdot a_1 \cdot a_2}{18} 1\]
c. Now say that person 1 and person 2 both know whether person 1 has a craving. Model this as a game and find Nash equilibria.

\[ \mathcal{E}U_1 (a_1^c, a_1^n, a_2^c, a_2^n) \]

\[ \mathcal{E}U_2 (a_1^c, a_1^n, a_2^c, a_2^n) \]

\[
\begin{align*}
\frac{d\mathcal{E}U_1}{da_1^c} &= \frac{1}{2} (-2a_1^c + 2a_2^c - 4a_1^c) = 0 \\
6a_1^c &= 2a_2^c \\
\left[ a_1^c = \frac{a_2^c}{3} \right]
\end{align*}
\]

\[
\begin{align*}
\frac{d\mathcal{E}U_2}{da_1^n} &= \frac{1}{2} (-2a_1^n + 2a_2^n - 2a_2^n) = 0 \\
4a_1^n &= 2a_2^n \\
\left[ a_1^n = \frac{a_2^n}{2} \right]
\end{align*}
\]

\[
\begin{align*}
\mathcal{E}U_1 (a_1^c, a_1^n, a_2^c, a_2^n) &= \frac{1}{2} (-a_1^c - a_2^c) - (a_2^c - 1)^2 + \frac{1}{2} (-a_1^n - a_2^n) - (a_2^n - 1)^2 \\
&= \frac{1}{2} \left( -a_1^c + 2a_2^c - (a_1^c)^2 - (a_2^c)^2 - 2a_2^c - 1 \right) \\
&\quad + \frac{1}{2} \left( -a_1^n + 2a_2^n - (a_1^n)^2 - (a_2^n)^2 - 2a_2^n - 1 \right)
\end{align*}
\]

\[
\begin{align*}
\frac{d\mathcal{E}U_1}{da_2^c} &= \frac{1}{2} (-2a_2^c + 2a_1^c - 2a_2^c + 2) = 0 \\
4a_2^c &= 2a_1^c + 2 \\
\left[ a_2^c = \frac{a_1^c + 2}{2} \right]
\end{align*}
\]

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\end{align*}
\]
\[ a_1^c = \frac{a_1^c}{3} \quad a_1^c = \frac{a_1^c + 1}{2} \]

\[ a_1^c = \frac{1}{3} \left( \frac{a_1^c + 1}{2} \right) \]

\[ b \cdot a_1^c = a_1^c + 1 \quad \Rightarrow \quad 5a_1^c = 1 \quad \Rightarrow \quad a_1^c = \frac{1}{5} \]

\[ a_2^c = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = \frac{3}{2} \quad \Rightarrow \quad a_2^c = \frac{3}{2} \]

\[ a_1^n = \frac{a_1^n}{2} \quad a_2^n = \frac{a_2^n + 1}{2} \]

\[ a_1^n = \frac{1}{2} \left( \frac{a_1^n + 1}{2} \right) \]

\[ 4a_1^n = a_1^n + 1 \quad \Rightarrow \quad 3a_1^n = 1 \quad \Rightarrow \quad a_1^n = \frac{1}{3} \]

\[ a_2^n = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = \frac{3}{2} \quad \Rightarrow \quad a_2^n = \frac{3}{2} \]

\[ a_c \quad a_1^n \quad a_2^n \]

\[ \frac{1}{5} \quad \frac{1}{3} \quad \frac{3}{5} \quad \frac{3}{3} \]

NE: \( \left( \frac{1}{5}, \frac{1}{3} \right) \quad \left( \frac{3}{5}, \frac{3}{3} \right) \)