This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, or any electronic or computational devices are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has two parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
1. Say that Liying (person 1) and Miguel (person 2) are roommates who are thinking about cooking some ramen. Each person can either cook (c) or not (n) a packet of ramen. However, they are uncertain about their own cooking skills. It is possible that they are both good cooks (G). But it is also possible that Liying is a bad cook (LB) or that Miguel is a bad cook (MB). All three possibilities are equally likely.

\[
\begin{array}{ccc|ccc}
 & c & n & & c & n & c & n \\
| c | 12, 12 & 6, 15 & | c | 12, 12 & 6, 3 & | c | 12, 12 & 6, 15 \\
| n | 15, 6 & 0, 0 & | n | 15, 6 & 0, 0 & | n | 3, 6 & 0, 0 \\
\end{array}
\]

\[G \quad LB \quad MB\]

Note that when both are good cooks, then both cooking ramen is pretty good (payoff 12) but the best thing is if the other person cooks and you don’t (payoff 15) because you get some of your roommate's ramen without having to do anything. When Liying is a bad cook, then the situation is the same, except for one change: if Liying cooks and Miguel doesn’t, then Miguel has to suffer eating Liying’s badly-cooked ramen (payoff 3). Similarly, when Miguel is a bad cook, if Miguel cooks and Liying doesn’t, then Liying suffers.

a. Say that both people are completely uninformed. Represent this as a strategic form game and find all Nash equilibria. (3 points)

b. Now say that Liying knows whether or not Miguel is a bad cook. However, Liying doesn’t know whether she herself is a bad cook. Miguel still is completely uninformed. Represent this as a strategic form game and find all Nash equilibria. (3 points)
Here are the payoffs again for your convenience.

\[
\begin{array}{ccc}
  & c & n \\
\hline
  c & 12,12 & 6,15 \\
n & 15,6 & 0,0 \\
\end{array}
\]

\[
\begin{array}{ccc}
  & c & n \\
\hline
  c & 12,12 & 6,3 \\
n & 15,6 & 0,0 \\
\end{array}
\]

\[
\begin{array}{ccc}
  & c & n \\
\hline
  c & 12,12 & 6,15 \\
n & 3,6 & 0,0 \\
\end{array}
\]

\[
\begin{array}{ccc}
  G & LB & MB \\
\end{array}
\]

c. Again, Liying knows whether or not Miguel is a bad cook but doesn’t know whether she herself is a bad cook. Now say that Miguel has the same information: Miguel knows whether or not he himself is a bad cook but does not know whether Liying is a bad cook. Represent this as a strategic form game and find all Nash equilibria. (4 points)

d. Finally, now say that both Liying and Miguel are both completely informed about everything. Find all Nash equilibria. (Do not write down an eight by eight game! You can do this in a simpler way.) (1 point)

e. Of all the situations above, which is the best for Liying? Which is best for Miguel? (1 point)
2. Say that person 1 and person 2 are on a blind date. Person 1 chooses a location \( a_1 \) and person 2 chooses a location \( a_2 \) on the beach, where \( a_1 \) and \( a_2 \) are both in the interval \([0, 1]\). The pier is at position 0 and the lifeguard station is at position 1. Person 1 likes to be close to person 2 but also the pier; person 2 likes to be close to person 1 but also the lifeguard station. Person 1’s utility function is given by \( u_1(a_1, a_2) = -\theta(a_1 - a_2)^2 - (a_1 - 0)^2 \) and person 2’s utility function is \( u_2(a_1, a_2) = -\theta(a_2 - a_1)^2 - (a_2 - 1)^2 \), where \( \theta \) depends on how much they like each other. If they are indifferent \((I)\), then \( \theta = 0 \). If they are fond of each other \((F)\), then \( \theta = 1 \). If they are crazy about each other \((C)\), then \( \theta = 2 \). Each of the three possibilities \( I, F, C \) are equally likely.

a. Say that neither person 1 nor person 2 knows anything about how much they like each other. Find the Bayesian Nash equilibrium of this game. (4 points)
b. Now say that person 1 always knows exactly how much they like each other. However, person 2 only knows if they are crazy about each other or not; person 2 cannot tell if they are indifferent or are just fond of each other. Find the Bayesian Nash equilibrium of this game. (4 points)

\[ \mathbb{E} u_1 = \frac{1}{3} \left( \frac{1}{2} (-a_1^F)^2 + \frac{1}{2} (\frac{1}{2} a_1^F - a_2^F)^2 - (a_1^F)^2 + \frac{1}{2} (-2(a_1^C - a_1^F) - (a_1^C))^2 \right) \]

\[ \frac{d\mathbb{E} u_1}{da_1^F} = \frac{1}{3} (-2a_1^F) = 0 \Rightarrow -2a_1^F = 0 \Rightarrow a_1^F = 0 \]

\[ \frac{d\mathbb{E} u_1}{da_1^C} = \frac{1}{3} (\frac{1}{2} (-2(a_1^F - a_2^F) - 2a_1^F) = 0 \Rightarrow \frac{-2(a_1^F - a_2^F) - 2a_1^F = 0}{-2a_1^F + 2a_2^F - 2a_1^F = 0} \Rightarrow 2a_2^F = 4a_1^F \]

\[ a_1^F = \frac{a_2^F}{2} \]

\[ a_1^C = \frac{2}{3} a_2^C \]

\[ \mathbb{E} u_2 = \frac{1}{3} (-a_2^C - 1)^2 + \frac{1}{2} (-a_2^C - a_2^F)^2 - (a_2^C - 1)^2 + \frac{1}{2} (-2(a_2^C - a_2^C) - (a_2^C))^2 \]

\[ \frac{d\mathbb{E} u_2}{da_2^F} = \frac{1}{3} (-2(a_2^C - 1) + \frac{1}{2} (-2(a_2^C - a_2^F) - 2(a_2^F - 1))) = 0 \]

\[ -2a_2^C + 2 - \frac{2a_2^C}{2} - 2a_2^F - 2a_2^F + 2 = 0 \]

\[ 2a_2^F + 4 = 6a_2^C \Rightarrow \frac{a_2^F}{a_2^C} = \frac{3}{2} \]

\[ \frac{d\mathbb{E} u_2}{da_2^C} = \frac{1}{3} (-4(a_2^C - a_2^C) - 2(a_2^C - 1)) = 0 \]

\[ -4a_2^C + 4a_2^C - 2a_2^C + 2 = 0 \]

\[ 4a_2^C + 2 = 6a_2^C \Rightarrow a_2^C = \frac{2a_2^C + 2}{3} \]

Solve for \( a_1^F, a_1^C, a_2^F, a_2^C \):

\[ a_1^F = \frac{a_1^F}{2} = \frac{1}{3} \left( \frac{a_2^F}{3} + \frac{1}{3} \right) \]

\[ 6a_2^F = 2a_1^F + 2 \]

\[ \frac{a_1^F}{a_2^F} = \frac{1}{3} \left( \frac{2a_2^F}{3} + \frac{1}{3} \right) \]

\[ \sum a_1 = 2 \]

\[ a_1^C = \frac{2}{3} \]

\[ \sum a_1^C = 2 \]

\[ a_2^C = \frac{2}{3} \left( \frac{a_2^C}{3} \right) + \frac{1}{3} = \frac{4}{15} + \frac{5}{15} = \frac{9}{15} = \frac{3}{5} \]

\[ a_2^C = \frac{3}{5} \]

NE: (0, \( \frac{2}{5}, \frac{2}{5} \)) , (\( \frac{4}{5}, \frac{4}{5} \), \( \frac{3}{5} \))
c. Now say that person 1 only knows whether they are indifferent or not; person 1 cannot tell if they are fond of each other or crazy about each other. Person 2 only knows if they are crazy about each other or not; person 2 cannot tell if they are indifferent or are just fond of each other. Find the Bayesian Nash equilibrium of this game. (4 points)

\[ E_{u_1} = \frac{1}{3} \left( -(a_{i, FC})^2 \right) + \frac{1}{3} \left( -(a_{i, FC} - a_{i, FC})^2 - (a_{i, FC})^2 \right) + \frac{1}{3} \left( -2 (a_{i, FC} - a_{i, FC})^2 - (a_{i, FC})^2 \right) \]

\[ \frac{dE_{u_1}}{da_{i, FC}} = \frac{1}{3} (-2a_{i, FC}) = 0 \Rightarrow a_{i, FC} = 0 \]

\[ \frac{dE_{u_1}}{da_{i, IF}} = \frac{1}{3} (-2 (a_{i, FC} - a_{i, IF}) - 2a_{i, FC}) + \frac{1}{3} (-4 (a_{i, FC} - a_{i, FC}) - 2a_{i, FC}) = 0 \]

\[ -2a_{i, FC} + 2a_{i, IF} - 2a_{i, FC} + 4a_{i, FC} - 2a_{i, FC} = 0 \]

\[ 2a_{i, IF} + 4a_{i, FC} = 10a_{i, FC} \Rightarrow a_{i, FC} = \frac{2}{5} + \frac{2a_{i, FC}}{5} \]

\[ E_{u_2} = \frac{1}{3} \left( -(a_{i, IF} - 1)^2 \right) + \frac{1}{3} \left( -(a_{i, IF} - a_{i, FC})^2 - (a_{i, IF} - 1)^2 \right) + \frac{1}{3} \left( -2 (a_{i, IF} - a_{i, FC})^2 - (a_{i, IF} - 1)^2 \right) \]

\[ \frac{dE_{u_2}}{da_{i, IF}} = \frac{1}{3} (-2(a_{i, IF} - 1)) + \frac{1}{3} (-2(a_{i, IF} - a_{i, FC}) - 2(a_{i, IF} - 1)) = 0 \]

\[ -2a_{i, IF} + 2a_{i, IF} - 2a_{i, IF} - 2a_{i, IF} + 2 = 0 \]

\[ 4 + 2a_{i, IF} = 6a_{i, IF} \Rightarrow a_{i, IF} = \frac{2}{3} + \frac{2a_{i, FC}}{3} \]

\[ \frac{dE_{u_2}}{da_{i, FC}} = \frac{1}{3} (-4 (a_{i, FC} - a_{i, FC}) - 2(a_{i, FC} - 1)) = 0 \]

\[ -4a_{i, FC} + 4a_{i, FC} - 2a_{i, FC} + 2 = 0 \]

\[ 4a_{i, FC} + 2 = 6a_{i, FC} \Rightarrow a_{i, FC} = \frac{2}{3} + \frac{2a_{i, FC}}{3} \]

Solve for \( a_{i, FC} \), \( a_{i, IF} \), \( a_{i, FC} \) :

\[ a_{i, FC} = \frac{1}{5} \left( \frac{2}{3} + \frac{2a_{i, FC}}{3} \right) + \frac{2}{5} \left( \frac{2}{3} + a_{i, FC} + \frac{1}{3} \right) = \frac{2}{15} + \frac{a_{i, FC}}{15} + \frac{4}{15} a_{i, FC} + \frac{2}{15} \]

\[ 10a_{i, FC} = 2 + a_{i, FC} + 4a_{i, FC} + 2 \]

\[ 4a_{i, FC} + 2 = 6a_{i, FC} \Rightarrow a_{i, FC} = \frac{2}{5} \]

\[ a_{i, IF} = \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} = \frac{10}{15} + \frac{2}{15} = \frac{12}{15} = \frac{4}{5} \]

\[ a_{i, FC} = \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} = \frac{10}{15} + \frac{2}{15} = \frac{12}{15} = \frac{4}{5} \]

\[ a_{i, FC} = \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} = \frac{10}{15} + \frac{2}{15} = \frac{12}{15} = \frac{4}{5} \]

So ME is

\[
\begin{pmatrix}
0 & \frac{2}{5} & \frac{2}{5} \\
\frac{4}{5} & \frac{4}{5} & \frac{3}{5}
\end{pmatrix}
\]