Homework 9     PS 30     November 2016

1. Say that Person 1 and Person 2 each decide whether to go to the auto racing match or the ballet. If they go to different places, both get utility zero. If they both go to the auto racing match, person 1 gets a utility of $4 and person 2 gets a utility of $1. If they both go to the ballet, person 1 gets a utility of $1 and person 2 gets a utility of $4. This is a “battle of the sexes” game. Now say that before playing this battle of the sexes game, person 1 can either burn $2 or not burn it. Person 2 can see whether person 1 burns the money or not.

a. Represent this as a strategic form game (each person has four strategies) and find all (pure strategy) Nash equilibria.

b. Show that by iteratively eliminating both strongly and weakly dominated strategies, one can predict that person 1 does not burn the money and that both go to the auto racing match.

2. Say that you and I are playing a game in which we both simultaneously yell out either $a$ or $b$. If we say the same letter, then you get $1 and I get nothing. If we say different letters, then I get $1 and you get nothing.

a. Model this as a game and find the Nash equilibrium.

b. Now say that I just had my wisdom teeth pulled out and my mouth is still quite numb. So it’s more difficult for me to say $b$: when I say $b$, I have to pay the cost $r$, where $r > 0$. Now we play the same game above (my payoffs have changed but yours have not). Model this as a game and find the Nash equilibrium.

c. How does my expected utility in the Nash equilibrium change as $r$ changes? What happens when $r = 1$? When $r > 1$?

3. Say that person 1 and person 2 are playing a drinking game which goes like this. There are $m$ beers in the refrigerator. Person 1 goes first by drinking either 1 or 2 beers. Then person 2 can drink either 1 or 2 beers. Then person 1 can drink either 1 or 2 beers, and so forth. In other words, when it is a person’s turn to drink, she can drink either 1 or 2 beers. Whoever drinks the last beer wins the game. Winning the game yields a payoff of 1 and losing yields a payoff of 0. However, there is an additional feature to the game: there is a “magic number” $x$ (which is greater than 0 and less than $m$). If after your turn, there are exactly $x$ beers left, then you lose the game and also have to go out and buy more beer; this has a payoff of −3 for the loser and a payoff of 1 for the winner.

a. Say that $m = 6$ and $x = 4$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

b. Say that $m = 6$ and $x = 3$. Model this as an extensive form game and find a subgame perfect Nash equilibrium.

c. Now let $m$ and $x$ be any number. Find a subgame perfect Nash equilibrium. For what values of $m$ and $x$ can person 1 guarantee a win? For what values of $m$ and $x$ can person 2 guarantee a win?
4. [from Spring 2002 final] There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability \( p \) and B with probability \( 1 - p \). Find \( p \).

5. [from Spring 2003 final] Say that we have 10 people. Each person is thinking about whether or not to join a revolt or not. Each person has a threshold: five people have threshold 4 and five people have threshold 6.

a. Find all pure strategy Nash equilibria of this game.

b. Say that you have some discount coupons which lower the cost of revolting and hence lower a person’s threshold. For example, if I give 3 coupons to a person with threshold 4, she now has threshold 1. If I give 1 coupon to a person with threshold 6, he now has threshold 5. By giving out coupons, I can change the game so that the only Nash equilibrium is one in which everyone revolts. I want to do this by giving out the fewest number of coupons. How do I distribute the coupons (who gets coupons, and how many does each person get)?

c. Now say that you can give tickets which raise the cost of revolting and hence raise a person’s threshold. For example, if I give 3 tickets to a person with threshold 4, she now has threshold 7. If I give 2 tickets to a person with threshold 6, he now has threshold 8. By giving out tickets, I can change the game so that the only Nash equilibrium is one in which no one revolts. I want to do this by giving out the fewest number of tickets. How do I distribute the tickets (who gets tickets, and how many does each person get)?

6. Say that we have three men, A, B, and C, and three women X, Y, and Z. Each person has preferences about which member of the opposite sex they would like to be matched with. For example, say woman X likes A the best, B second best, and C worst. Assume that there are no ties (i.e. it cannot be that woman Y likes A the best and B and C are tied for worst). Note that there are six possible matchings.

a. Is it possible for people’s preferences to be such that there exists exactly one stable matching? If so, write down the preferences which make this possible. If not, explain why not.

b. Is it possible for people’s preferences to be such that there exists exactly two stable matchings? If so, write down the preferences which make this possible. If not, explain why not.

c. Is it possible for people’s preferences to be such that there exists exactly three stable matchings? If so, write down the preferences which make this possible. If not, explain why not.
7. Say that there are five people choosing among three candidates \( x, y, z \). Persons 1 and 2’s preference ordering from best to worst is \( x, z, y \). Persons 3 and 4 have preference ordering \( y, z, x \). Person 5 has preference ordering \( z, x, y \).

a. Say that they make their decision using a runoff procedure: first everyone votes for their first choice, and the two alternatives which get the most votes go to a runoff. In the runoff, each person votes for one of these two candidates, and whoever gets the most votes in the runoff wins. Which candidate wins in this procedure?

b. Is the candidate who wins the runoff the Condorcet winner? Is there a Condorcet winner?

8. Say that there are three people choosing among five candidates \( s, t, w, x, y \). Person 1’s preference ordering from best to worst is \( w, x, t, y, s \). Person 2’s preference ordering is \( y, w, x, t, s \). Person 3’s preference ordering is \( s, x, t, y, w \).

a. Say that people decide using majority rule according to some agenda. For example, one agenda might be to vote on \( w \) first; if \( w \) loses, then vote on \( x \); if \( x \) loses, then vote on \( s \); if \( s \) loses, then vote on \( t \) versus \( y \). Can you find an agenda in which \( t \) is chosen?

b. What is the top cycle? Remember that the candidates in the top cycle are those which win given some suitably chosen agenda. Candidates not in the top cycle are those which are never chosen regardless of the agenda.

9. Say that the town of Mollusk Beach is deciding how many oil refineries to build. Forty percent of voters prefer no refineries, 36 percent prefer 1 refinery, and 24 percent prefer two refineries. Say that there are two candidates A and B, and each has to take a position on this issue. Given their positions, each voter will vote for the candidate whose position is closest to their own (if there are two candidates who are equally far away, assume that the vote is split equally among the two candidates). Each candidate wants to maximize the total number of votes she receives.

a. Say that candidates A and B choose their positions simultaneously. Which positions will they take?

b. Say now that candidate A can choose her position first and then candidate B chooses her position. Which positions will they take?

c. Now say that there are three candidates, A, B, and C. Say that candidate A chooses her position first, then candidate B, and then finally candidate C. Which positions will they take?