This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, or any electronic or computational devices are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has two parts. Each part is weighted equally (12 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
1. It is Valentine’s Day and Person 1 and Person 2 are each considering whether to send flowers ($f$) or not ($n$) to Pat (Pat is not a player). Pat likes either Person 1 or Person 2 (both possibilities are equally likely), but Person 1 and Person 2 do not necessarily know whom Pat likes. The payoffs are given below.

\[
\begin{array}{ccc}
& f & n \\
\hline
f & 8, -2 & 8, 0 \\
n & 0, -10 & 0, 0 \\
\end{array}
\]

\[
\begin{array}{ccc}
& f & n \\
\hline
f & -12, 8 & -10, 0 \\
n & 0, 8 & 0, 0 \\
\end{array}
\]

Pat likes 1

Pat likes 2

Note that if you do not send flowers, you get a payoff of 0. If Pat likes you and you send flowers to Pat, you get a payoff of 8. If Pat does not like you and you are the only person who sends flowers, you are embarrassed and get a payoff of −10. If Pat likes Person 2 and both people send flowers, then Person 1 is super embarrassed for seeming to compete with Person 2, and gets a payoff of −12. If Pat likes Person 1 and both people send flowers, then Person 2 gets payoff −2; Person 2 is a little embarrassed but not terribly so because Person 2 can claim that they were sent in the spirit of friendship, since both people sent flowers.

a. Say that both people are completely uninformed about whom Pat likes. Represent this as a strategic form game and find all Nash equilibria. (2 points)

b. Now say that both Person 1 and Person 2 know whom Pat likes. Represent this as a strategic form game and find all Nash equilibria. (2 points)
Here are the payoffs again for your convenience.

\[
\begin{array}{ccc}
 f & n \\
 f & 8, -2 & 8, 0 \\
n & 0, -10 & 0, 0 \\
\end{array}
\quad
\begin{array}{ccc}
 f & n \\
 f & -12, 8 & -10, 0 \\
n & 0, 8 & 0, 0 \\
\end{array}
\]

Pat likes 1

Pat likes 2

c. Now say that Person 1 knows whom Pat likes but Person 2 does not know. Represent this as a strategic form game and find all Nash equilibria. (3 points)

d. Now say that Person 2 knows whom Pat likes but Person 1 does not know. Represent this as a strategic form game and find all Nash equilibria. (3 points)

e. Say that Person 1 does not know whom Pat likes. Would Person 1 prefer that Person 2 know whom Pat likes or would Person 1 prefer that Person 2 not know? (1 point)

f. Say that Person 2 does not know whom Pat likes. Would Person 2 prefer that Person 1 know whom Pat likes or would Person 2 prefer that Person 1 not know? (1 point)
2. Say that person 1 and person 2 are friends who like to use the spinning bikes at the gym. Person 1 chooses to exercise for $a_1$ minutes and person 2 chooses $a_2$ minutes. The only problem is that person 1 sometimes gets distracted by one of the gym employees. If the employee is not there, then person 1’s utility function is given by $u_1(a_1, a_2, N) = (40 + a_2 - a_1)a_1$. In other words, if the employee is not there, everything is fine and person 1’s enjoyment of exercise increases when person 2 exercises more. However, if the employee is there, then person 1’s utility function is given by $u_1(a_1, a_2, E) = (40 - a_2 - a_1)a_1$. If the employee is there, then person 1’s enjoyment of exercise decreases when person 2 exercises more, because person 1 does not want to look bad in comparison with person 2 when the employee is around. Person 2 is never distracted by the employee and person 2’s utility function is $u_2(a_1, a_2) = (40 + a_1 - a_2)a_2$, regardless of whether the employee is there or not. The probability that the employee is there is $1/2$.

a. Say that neither person 1 nor person 2 knows whether the employee is there. Find the Bayesian Nash equilibrium of this game. (3 points)
b. Now say that both person 1 and person 2 know whether the employee is there. Find the Bayesian Nash equilibrium of this game. The utility functions are shown again here for your reference. (3 points)

\[ u_1(a_1, a_2, N) = (40 + a_2 - a_1)a_1 \]
\[ u_1(a_1, a_2, E) = (40 - a_2 - a_1)a_1 \]
\[ u_2(a_1, a_2) = (40 + a_1 - a_2)a_2 \]
c. Now say that person 1 knows whether the employee is there but person 2 does not. Find the Bayesian Nash equilibrium of this game. The utility functions are shown again here for your reference. (4 points)

\[
\begin{align*}
    u_1(a_1, a_2, N) &= (40 + a_2 - a_1)a_1 \\
    u_1(a_1, a_2, E) &= (40 - a_2 - a_1)a_1 \\
    u_2(a_1, a_2) &= (40 + a_1 - a_2)a_2
\end{align*}
\]
d. Say that person 1 knows whether the employee is there. Does person 1 prefer that person 2 know that the employee is there, or does person 1 prefer that person 2 not know? (2 points)