1. Say that two firms are in a Cournot duopoly. Firm 1 produces \( q_1 \) and Firm 2 produces \( q_2 \). Given total production \( q_1 + q_2 \), the good sells at price \( 140 - (q_1 + q_2) \). Firm 1 has no production costs and hence its utility is given by \( u_1(q_1, q_2) = (140 - (q_1 + q_2))q_1 \). However, Firm 2 has production costs of \( (q_2)^2 \) and hence its utility is given by \( u_2(q_1, q_2) = (140 - (q_1 + q_2))q_2 - (q_2)^2 \).

a. Find the Nash equilibrium of this game.

b. Can you find some strategies which are dominated for Firm 1? For Firm 2?

2. Say that two countries are at war. Country 1 spends \( a_1 \) on the military and Country 2 spends \( a_2 \). Given this, the probability that Country 1 wins is \( \frac{a_1}{a_1 + a_2} \) and the probability that Country 2 wins is \( \frac{a_2}{a_1 + a_2} \). Winning the war is worth 16 to both countries. Hence Country 1’s utility function is

\[ u_1(a_1, a_2) = 16 \frac{a_1}{a_1 + a_2} - a_1 \]

and Country 2’s utility function is

\[ u_2(a_1, a_2) = 16 \frac{a_2}{a_1 + a_2} - a_2. \]

a. Find the Nash equilibria of this game.

b. Now say that Country 2 suddenly realizes much more is at stake and winning the war is now worth 48 to Country 2. Winning the war is still worth 16 for Country 1. What is Country 2’s new utility function? How does the Nash equilibrium change? xs

3. Say you’re at the Clippers-Lakers game at the Staples Center and you can choose either to yell at the top of your lungs like a maniac or to sit quietly and watch the game. If you make \( n \) minutes of noise and watch the game for \( w \) minutes, your utility is given by \( u(n, w) = nw \). Since the game is 60 minutes long, you have the constraint \( n + w = 60 \).

a. Say that you are the only fan in the audience. What is your optimal choice of \( n \) and \( w \)?

b. Now say that there are two fans in the audience. Now the total amount of noise \( n \) is given by \( n = n_1 + n_2 \). Person 1’s utility is \( u_1(n, w_1) = nw_1 \) and person 2’s utility is \( u_2(n, w_2) = nw_2 \). Of course, we have the constraints \( n_1 + w_1 = 60 \) and \( n_2 + w_2 = 60 \). Model this as a strategic form game and find the (pure strategy) Nash equilibrium. Is the Nash equilibrium Pareto efficient (can you both become better off by not playing the Nash equilibrium)?

c. Now say that there are \( m \) identical fans in the audience (each with the same utility function given above). There is a Nash equilibrium in which everyone cheers an identical amount. Find it. Does the total amount of cheering approach 60 as \( m \) increases?
4. Say that Cruz and Rubio each have 60 million dollars to spend in three primaries: North Carolina, Georgia, and Florida. North Carolina has 10 delegates, Georgia has 20, and Florida has 30. If a candidate spends more money in a state than his opponent, then he gets all the delegates. If the candidates spend equal amounts of money in a state, then they split the state’s delegates. For example, if Cruz spends 5 million in North Carolina, 15 million in Georgia, and 40 million in Florida, which we can write as (5, 15, 40), and Rubio spends (15, 25, 20), then Cruz wins Florida and gets 30 delegates while Rubio wins North Carolina and Georgia and gets \(10 + 20 = 30\) delegates.

a. Are there pure strategy Nash equilibria in this game? If so, find them.

b. Now say that Cruz has 80 million dollars while Rubio still has 60 million. Are there pure strategy Nash equilibria in this game? If so, find them.