1. Ann and Bob are each trying to win a prize in a school raffle (lottery). Each can buy either 0, 1, 2, or 3 raffle tickets. Ann and Bob are the only two people in the raffle, and each ticket has an equal chance of winning. So for example, if Ann buys 2 tickets and Bob buys 3 tickets, then Ann has a 2/5 chance of winning and Bob has a 3/5 chance of winning (if no one buys any tickets, the raffle is cancelled). The prize is worth $60, and both Ann and Bob care about their “expected payoffs”: for example, if Ann has a 2/5 chance of winning, her expected payoff is $24. Model the following situations with strategic form games.

a. Say that raffle tickets are free. What does the game look like? Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?

b. Now say that raffle tickets cost $6 each. What does the game look like? Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?

c. Now say that raffle tickets cost $10 each. What does the game look like? Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?

d. Now say that the people running the raffle choose how much raffle tickets cost. They want to make as much money as possible. Assume that given the cost of raffle tickets, Ann and Bob play the Nash equilibrium which is most favorable to the people running the raffle (in other words, if there is more than one Nash equilibrium, assume that they play the one in which the most tickets are sold). Which price should the people running the raffle choose? Is it possible for them to sell more than $60 worth of tickets?

2. Say that persons 1, 2, and 3 each decide whether to go to restaurant A or restaurant B. Person 1 wants the dinner group to be as large as possible. For person 1, the worst thing is if she goes to a restaurant alone, the best thing is if all three go to the same place, and going with one person (it doesn’t matter which) is OK, neither best or worst. Person 2 is the exact opposite; she wants the dinner group to be as small as possible. All person 3 cares about is going to the same place as person 1, since he likes person 1.

a. Model this as a strategic form game.

b. Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?

c. Now say that person 3 loses interest in person 1 and becomes grouchy like person 2. Model this as a strategic form game.

d. Are any strategies in this game strongly or weakly dominated? Can you iteratively eliminate strongly or weakly dominated strategies? What are the (pure strategy) Nash equilibria of this game?
3. In a simplified version of “Battleship,” say that there are four spaces, numbered 1, 2, 3, 4. Person 1 chooses to fire a missile at one of these four spaces. Person 2 has a ship which is two spaces long, and chooses where to put the ship on the board: she can either put it on spaces 1 and 2, on spaces 2 and 3, or on spaces 3 and 4. The people make their choices simultaneously.

a. Model this as a strategic form game and use the method iterative elimination of (strongly or weakly) dominated strategies to eliminate as many strategies as possible (i.e. keep on eliminating until you can’t eliminate any more).

b. Find all mixed-strategy and pure-strategy Nash equilibria of the remaining game.

c. Now say that there are 5 spaces, numbered 1, 2, 3, 4, 5. Model this as a strategic form game and find all mixed-strategy and pure-strategy Nash equilibria. Make a prediction in this game like you did before (iteratively eliminate dominated strategies, and then find pure strategy or mixed strategy Nash equilibria of the remaining game).

d. Now say that there are \(m\) spaces in a line, numbered 1, 2, 3, \ldots, \(m\). Can you make a prediction in this game?

e. Now say that they play on a board with five squares, as shown below.

   
   
Again, player 1 fires a missile into one of the five squares and thus she has five possible strategies. Player 2 places a battleship on the board. The battleship occupies two adjacent squares and can be oriented vertically or horizontally. Thus player 2 has five possible strategies. Make a prediction in this game.

4. Say that two firms are in a Cournot duopoly. Firm 1 produces \(q_1\) and Firm 2 produces \(q_2\). Given total production \(q_1 + q_2\), the good sells at price \(140 - (q_1 + q_2)\). Firm 1 has no production costs and hence its utility is given by \(u_1(q_1, q_2) = (140 - (q_1 + q_2))q_1\). However, Firm 2 has production costs of \((q_2)^2\) and hence its utility is given by \(u_2(q_1, q_2) = (140 - (q_1 + q_2))q_2 - (q_2)^2\).

a. Find the Nash equilibrium of this game.

b. Can you find strategies which are dominated for Firm 1? For Firm 2?
5. Say that two countries are at war. Country 1 spends $a_1$ on the military and Country 2 spends $a_2$. Given this, the probability that Country 1 wins is $\frac{a_1}{a_1+a_2}$ and the probability that Country 2 wins is $\frac{a_2}{a_1+a_2}$. Winning the war is worth 16 to both countries. Hence Country 1’s utility function is

$$u_1(a_1, a_2) = 16 \frac{a_1}{a_1+a_2} - a_1$$

and Country 2’s utility function is

$$u_2(a_1, a_2) = 16 \frac{a_2}{a_1+a_2} - a_2.$$

a. Find the Nash equilibria of this game.

b. Now say that Country 2 suddenly realizes much more is at stake and winning the war is now worth 48 to Country 2. Winning the war is still worth 16 for Country 1. What is Country 2’s new utility function? How does the Nash equilibrium change?

6. Say that two students are studying. Student 1 chooses to study $a_1$ hours and Student 2 chooses to study $a_2$ hours. Student 1 likes studying with student 2, but student 2 doesn’t like studying with student 1. Hence Student 1’s utility function is

$$u_1(a_1, a_2) = (10 + a_2)a_1 - (a_1)^2$$

but Student 2’s utility function is

$$u_2(a_1, a_2) = (10 - a_1)a_2 - (a_2)^2.$$

a. Find the Nash equilibria of this game.

7. Say you’re at the Clippers-Lakers game at the Staples Center and you can choose either to yell at the top of your lungs like a maniac or to sit quietly and watch the game. If you make $n$ minutes of noise and watch the game for $w$ minutes, your utility is given by $u(n, w) = nw$. Since the game is 60 minutes long, you have the constraint $n + w = 60$.

a. Say that you are the only fan in the audience. What is your optimal choice of $n$ and $w$?

b. Now say that there are two fans in the audience. Now the total amount of noise $n$ is given by $n = n_1 + n_2$. Person 1’s utility is $u_1(n, w_1) = nw_1$ and person 2’s utility is $u_2(n, w_2) = nw_2$. Of course, we have the constraints $n_1 + w_1 = 60$ and $n_2 + w_2 = 60$. Model this as a strategic form game and find the (pure strategy) Nash equilibrium. Is the Nash equilibrium Pareto efficient (can you both become better off by not playing the Nash equilibrium)?

c. Now say that there are $m$ identical fans in the audience (each with the same utility function given above). There is a Nash equilibrium in which everyone cheers an identical amount. Find it. Does the total amount of cheering approach 60 as $m$ increases?
8. Say that Romney and Gingrich each have 60 million dollars to spend in three primaries: North Carolina, Georgia, and Florida. North Carolina has 10 delegates, Georgia has 20, and Florida has 30. If a candidate spends more money in a state than his opponent, then he gets all the delegates. If the candidates spend equal amounts of money in a state, then they split the state’s delegates. For example, if Romney spends 5 million in North Carolina, 15 million in Georgia, and 40 million in Florida, which we can write as (5, 15, 40), and Gingrich spends (15, 25, 20), then Romney wins Florida and gets 30 delegates while Gingrich wins North Carolina and Georgia and gets 10 + 20 = 30 delegates.

a. Are there pure strategy Nash equilibria in this game? If so, find them.

b. Now say that Romney has 80 million dollars while Gingrich still has 60. Are there pure strategy Nash equilibria in this game? If so, find them.

9. Say you have an admirer whom you don’t like very much. You can either go to the library or the coffee shop to study. You prefer the coffee shop but you want to avoid your admirer. Your admirer can also go to the library or coffee shop to study. Your admirer prefers the library but wants to be where you are more than anything else. So the game looks like:

<table>
<thead>
<tr>
<th></th>
<th>You go to library</th>
<th>You go to coffeeshop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admirer goes to library</td>
<td>0, 3 4, 0</td>
<td></td>
</tr>
<tr>
<td>You go to coffeeshop</td>
<td>6, 0</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

a. Find all (pure strategy and mixed strategy) Nash equilibria of this game.

b. Now say that you begin to actually enjoy your admirer’s company. The game is now:

<table>
<thead>
<tr>
<th></th>
<th>You go to library</th>
<th>You go to coffeeshop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admirer goes to library</td>
<td>4, 3 0, 0</td>
<td></td>
</tr>
<tr>
<td>You go to coffeeshop</td>
<td>0, 0</td>
<td>6, 1</td>
</tr>
</tbody>
</table>

Find all (pure strategy and mixed strategy) Nash equilibria of this game.

10. Say we have two basketball players, Nash and Yao. Nash has the ball 20 feet from the basket and Yao is defending. Nash can either take a jump shot or drive the basket. Yao can either come out and contest the shot, or can stay in and protect the basket. The game looks like this.

<table>
<thead>
<tr>
<th></th>
<th>Yao comes out</th>
<th>Yao protects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash takes jump shot</td>
<td>0.3, 0.7</td>
<td>0.4, 0.6</td>
</tr>
<tr>
<td>Nash drives</td>
<td>0.5, 0.5</td>
<td>0.3, 0.7</td>
</tr>
</tbody>
</table>

For example, if Nash takes a jump shot and Yao comes out, then Nash has a 30 percent chance of scoring and Yao has a 70 percent chance of successfully defending.

a. Find the mixed strategy Nash equilibrium of this game. In the mixed Nash equilibrium, what is Nash’s overall probability of scoring?

b. Now say that Nash practices over the summer and improves his jump shot. Now the game looks like this.
<table>
<thead>
<tr>
<th></th>
<th>Yao comes out</th>
<th>Yao protects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash takes jump shot</td>
<td>0.4, 0.6</td>
<td>0.5, 0.5</td>
</tr>
<tr>
<td>Nash drives</td>
<td>0.5, 0.5</td>
<td>0.3, 0.7</td>
</tr>
</tbody>
</table>

Find the mixed strategy Nash equilibrium of this new game. In the mixed Nash equilibrium, what is Nash’s overall probability of scoring?

c. After Nash improves his jump shot, does he go to his jump shot more often? After Nash improves his jump shot, is it more successful? After Nash improves his jump shot, is his drive more successful? Why does an improved jump shot improve other parts of Nash’s game?