1. \( A = \{ 0, 2, 4, 5, 8 \} \) \( u(a) = -5^a \)

<table>
<thead>
<tr>
<th>( a )</th>
<th>( u(a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-10</td>
</tr>
<tr>
<td>4</td>
<td>-20</td>
</tr>
<tr>
<td>6</td>
<td>-30</td>
</tr>
<tr>
<td>8</td>
<td>-40</td>
</tr>
</tbody>
</table>

2. \( A = \{ 0, 2, 4, 6, 8 \} \) \( u(a) = (a-3)^2 \)

<table>
<thead>
<tr>
<th>( a )</th>
<th>( u(a) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
</tr>
</tbody>
</table>

3. \( A = \{ 0, 2, 4, 6, 8 \} \) \( u(a) = -(a-3)^2 \)

<table>
<thead>
<tr>
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</tr>
<tr>
<td>8</td>
<td>-25</td>
</tr>
</tbody>
</table>
4. \( A = [0, 8) \) \( u(a) = -5a \)

\[ u(a) = -5a \]

\( a = 0 \) maximizes utility.

5. \( A = [0, 8) \) \( u(a) = (a-3)^2 \)

\[ u(a) = (a-3)^2 \]

\( a = 8 \) maximizes utility.

Note that the first order method does not work in this example.
6. \( A = [0, 6] \) \( u(a) = -(a-3)^2 \)

\[ u(a) = -(a^2 - 6a + 9) \]

\[ = -a^2 + 6a - 9 \]

\[ u(a) \]

\[ \frac{dy}{da} = -2a + 6 = 0 \]

\[ 6 = 2a \]

\[ a = 3 \]

We can use the first-order method to find the optimum \( a \).

7. \( A = [0, 6] \) \( u(a) = a^3 - 6a^2 + 12a \)

\[ a = 6 \] maximizes utility.

\[ \frac{dy}{da} = 3a^2 - 12a + 12 = 0. \]

\[ a^2 - 4a + 4 = 0 \]

\[ (a-2)(a-2) = 0 \]

\[ a = 2 \]

Can we use first-order method?

The derivative of \( U \) at \( a = 2 \) is zero, but \( a = 2 \) is not a maximum. The first-order method does not work.
8. \( A = [0, 4] \) \( U(a) = a^2 - 9a + 15 \\
\frac{du}{da} = 3a^2 - 18a + 15 = 0 \\
a^2 - 6a + 5 = 0 \\
(a-1)(a-5) = 0. \\
a = 1 \text{ or } a = 5 \\
5 \text{ is not in } A \\
so \text{ it cannot be chosen.} \\
\text{So } a = 1 \text{ maximum utility.}

9. \( A = [0, 6] \) \( U(a) = a^2 - 9a + 15 \\
\frac{du}{da} = 3a^2 - 18a + 15 = 0 \\
a^2 - 6a + 5 = 0 \\
(a-1)(a-5) = 0 \\
a = 1 \text{ or } a = 5 \\
\text{But from the graph, } a = 5 \text{ is a minimum, not a maximum.} \\
\text{So } a = 1 \text{ maximum utility.}
10. \( A = [0, 10] \) \( u(a) = a^3 - 9a^2 + 15a \)

\[
\frac{du}{da} = 3a^2 - 18a + 15 = 0
\]

\[a^2 - 6a + 5 = 0 \]

\[(a - 1)(a - 5) = 0\]

\[a = 1 \quad a = 5\]

But from the graph, \( a = 1 \) is a local maximum

\[a = 5 \text{ is a minimum.}\]

\[a = 10 \text{ maximizes utility.} \]

The first-order method does not work.