1. a

\[ u_1(s_1, s_2) = (140 - (s_1 + s_2)) s_1 \]

\[ u_2(s_1, s_2) = (140 - (s_1 + s_2)) s_2 - (s_2)^2 \]

1's best response?

Expand to get

\[ u_1(s_1, s_2) = 140 s_1 - (s_1)^2 - s_1 s_2 \]

\[ \frac{du_1}{ds_1} = 140 - 2s_1 - s_2 \]

Set derivative equal to zero

\[ 0 = 140 - 2s_1 - s_2 \]

\[ 2s_1 = 140 - s_2 \]

\[ s_1 = 70 - \frac{s_2}{2} \] 1's best response

2's best response?

\[ u_2(s_1, s_2) = 140 s_2 - s_1 s_2 - (s_2)^2 - (s_1)^2 \]

\[ \frac{du_2}{ds_2} = 140 - s_1 - 2s_2 - 2s_2 \]

\[ 0 = 140 - s_1 - 4s_2 \]

\[ 4s_2 = 140 - s_1 \]

\[ s_2 = 35 - \frac{s_1}{4} \] 2's best response

\[ 2 \] \[ 1 \] \[ \text{the NE is where the best responses intersect.} \]

\[ s_1 = 70 - \frac{s_2}{2} \] 1's best response

\[ s_2 = 35 - \frac{s_1}{4} \] 2's best response

\[ \text{solve new equation:} \]

\[ 2s_1 = 140 - 35 + \frac{s_1}{4} \]

\[ \frac{7}{4} s_1 = 105 \]

\[ s_1 = 60 \]

\[ s_2 = 35 - \frac{60}{4} = 20 \]

\( s_0 (60, 20) \) is the Nash eq.
16. Since \( U_1(\bar{z}_1, \bar{z}_2) = (140 - (\bar{z}_1 + \bar{z}_2)) \bar{z}_1 \)

we have \( U_1(0, \bar{z}_2) = 0 \) for any \( \bar{z}_2 \).

So I can always get utility 0 by playing \( \bar{z}_1 = 0 \).

So \( \bar{z}_1 = 150 \) (for example) is strongly dominated by \( \bar{z}_1 = 0 \)

because \( U_1(150, \bar{z}_2) \) is negative for any \( \bar{z}_2 \).

\[ u_1(a_1, a_2) = 16 \frac{a_1}{a_1 + a_2} - a_1 \]

\[ u_2(a_1, a_2) = 16 \frac{a_2}{a_1 + a_2} - a_2 \]

1's best response?

\[ \frac{du_1}{da_1} = 16 \frac{(a_1 + a_2) - a_1}{(a_1 + a_2)^2} - 1 = 0 \]

\[ 16 a_1 = (a_1 + a_2)^2 \Rightarrow 4 \sqrt{a_1} = a_1 + a_2 \Rightarrow a_1 = 4 \sqrt{a_1} - a_2 \]

2's best response?

\[ \frac{du_2}{da_2} = 16 \frac{(a_1 + a_2) - a_2}{(a_1 + a_2)^2} - 1 = 0 \]

\[ 16 a_2 = (a_1 + a_2)^2 \Rightarrow 4 \sqrt{a_2} = a_1 + a_2 \Rightarrow a_2 = 4 \sqrt{a_2} - a_1 \]

Nash equilibria: We know

\[ 16 a_2 = (a_1 + a_2)^2 \]

and

\[ 16 a_1 = (a_1 + a_2)^2 \]

Thus

\[ 16 a_2 = 16 a_1 \Rightarrow a_1 = a_2 \]

So

\[ 16 a_1 = (2a_1)^2 \]

\[ 16 a_1 = 4(a_1)^2 \]

\[ 4 = \boxed{a_1} \] and \[ \boxed{4 = a_2} \]

So NE is \((4, 4)\).
2b. Now  \[ U_i(a_i, a_2) = 16 \frac{a_i}{a_i + a_2} - a_2, \]

\[ U_2(a_1, a_2) = 48 \frac{a_1}{a_1 + a_2} - a_2 \]

Person 1's best response is \( a_1 = 0 \) because \( 16 a_2 = (a_1 + a_2)^2 \) \( \Rightarrow a_1 = 4 \sqrt{a_2} - a_2 \)

Person 2's best response?

\[ \frac{dU_2}{da_2} = 48 \frac{(a_1 + a_2) - a_2}{(a_1 + a_2)^2} - 1 = 0 \]

\[ 48 a_1 = (a_1 + a_2)^2 \Rightarrow a_2 = \sqrt{48} a_1 - a_1 \]

To find NE, we need

\[ 16 a_2 = (a_1 + a_2)^2 \]

\[ 48 a_1 = (a_1 + a_2)^2 \]

So

\[ 16 a_2 = 48 a_1 \]

\[ a_2 = 3 a_1 \]

So

\[ 48 a_1 = (a_1 + 3a_1)^2 \]

\[ 48 a_1 = (4a_1)^2 \]

\[ 48 a_1 = 16(a_1)^2 \]

\[ 3 a_1 = (a_1)^2 \Rightarrow a_1 = 3 \]

\[ a_2 = 3 \cdot 3 = 9 \]

So, NE is \( (3, 9) \).

Person 2 cares more about winning and hence spends much on warfare. Hence Person 1 weeks off.
4. \( u(n, w) = nw \)
\[ n + w = 60 \quad \Rightarrow \quad n = 60 - w \]

a. \( u'_1(w) = (60 - w)w = 60w - w^2 \)
\[ 0 = \frac{d u}{d w} = 60 - 2w \]
\[ 0 = 60 - 2w \quad \Rightarrow \quad 2w = 60 \quad \Rightarrow \quad w = 30 \]
\[ n = 60 - 30 = 30 \]

b. \( u_1(n, w_1) = nw_1 \)
\( u_2(n, w_2) = nw_2 \)
\( n = n_1 + n_2 \)
\( n_1 + w_1 = 60 \quad \Rightarrow \quad n_1 = 60 - w_1 \)
\( n_2 + w_2 = 60 \quad \Rightarrow \quad n_2 = 60 - w_2 \)

\( u_1(w_1, w_2) = (n_1 + n_2)w_1 = (60 - w_1 + 60 - w_2)w_1 = (120 - w_1 - w_2)w_1 \)
\( u_2(w_1, w_2) = (120 - w_1 - w_2)w_2 \) (similarly).

\[ \frac{d u_1}{d w_1} = (120 - 2w_1 - w_2 = 0 \quad \Rightarrow \quad 120 - w_2 = 2w_1 \quad \Rightarrow \quad w_1 = 60 - \frac{w_2}{2} \] (best response)

\[ \frac{d u_2}{d w_2} = (120 - w_1 - 2w_2 = 0 \quad \Rightarrow \quad w_2 = 60 - \frac{w_1}{2} \] (2/3 best response)

So \( w_1 = 60 - \frac{1}{2} \left(60 - \frac{w_2}{2}\right) \)
\[ 2w_1 = 120 - \left(60 - \frac{w_2}{2}\right) \]
\[ \frac{3}{2}w_1 = 60 \quad \Rightarrow \quad w_1 = 40 \]
\[ w_2 = 60 - \frac{w_2}{2} = 40 \]

5. \( \text{NE is } (40, 40) \)
\[ u_i(w_1, w_2, \ldots, w_m) = (60 - w_1 + 60 - w_2 + \cdots + 60 - w_m) w_i \]

\[ u_2(\quad \quad \quad ) = \quad \quad \quad w_2 \]

\[ \frac{\partial u_1}{\partial w_1} = -w_1 + (60 - w_1 + 60 - w_2 + \cdots + 60 - w_m) = 0 \implies w_1 = 60 - w_1 + 60 - w_2 + \cdots + 60 - w_m \]

\[ \frac{\partial u_2}{\partial w_2} = -w_2 + (\quad \quad \quad ) = 0 \implies w_2 = 60 - w_1 + 60 - w_2 + \cdots + 60 - w_m \]

\[ \text{Here } w_1 = w_2 = \ldots = w_m. \]

So \[ w_1 = (60 - w_1 + 60 - w_2 + \cdots + 60 - w_m) \]

\[ w_1 = 60m - mw_1 \]

\[ (m+1)w_1 = 60m \]

\[ w_1 = \frac{60}{m+1} \]

\[ w_2 = w_3 = \ldots = w_m = \frac{60}{m+1} \]

So the NE is \( (\frac{60}{m+1}, \frac{60}{m+1}, \ldots, \frac{60}{m+1}) \)

\[ \sin u \quad w_1 = 60 \cdot \frac{60}{m+1} \]

\[ \sin u \quad w_1 = 60 - 60 \cdot \frac{m}{m+1} = \frac{60m}{m+1} = \frac{60}{m+1} \]

So as \( m \) grows large, each person makes less noise.

But the total amount of noise is \( \frac{60}{m+1} \cdot m \)

Which approaches \( 60 \) (the entire game) as \( m \) grows large.
If you spend $60 or in Florida, either you win Florida
(if your opponent spends less than $40) or you tie everywhere
(if your opponent also spends $60 in Florida).

Either way, you get 30 delegates at least.

So there is no NE in which someone gets less than 30 delegates.

Since there are 60 delegates total, in any NE, each person gets 30.

The only way this can happen is if they tie in all states
or if the candidate wins Florida and one wins NC and GA.

In this case, for example

\[
\begin{array}{c|c|c}
\text{NC} & \text{F} & \text{GA} \\
\hline
0 & 0 & 60 \\
\end{array}
\]

Then this is not a NE (the person winning F can win another state and still)

\[
\begin{array}{c|c|c|c}
\text{F} & \\
\hline
60 & \\
\end{array}
\]

If they tie in all states, for example

\[
\begin{array}{c|c|c}
\text{NC} & \text{F} & \text{GA} \\
\hline
10 & 20 & 30 \\
\end{array}
\]

Then this is not a NE because one candidate can win F by eliminating

so the only NE is if F cannot be eliminated. (I.e.,

\[
\begin{array}{c|c|c}
\text{F} & \\
\hline
60 & \\
\end{array}
\]
Now there is no NE because Romney can always put more money on all states than Gingrich.

Hence in any NE, Romney gets all 60 delegates and Gingrich 0.

However, Gingrich can always get at least some delegates by putting all his money on one state. (Romney can’t afford to put 60 on all three states).

Hence there is no pure strategy NE.
3. \( u_1(a_1, a_2) = (10 + a_2) a_1 - (a_1)^2 \)
\( u_2(a_1, a_2) = (10-a_1) a_2 - (a_2)^2 \)

**Best response for 1**:
\[
\frac{du_1}{da_1} = 10 + a_2 - 2a_1 = 0
\]
\[
2a_1 = 10 + a_2
\]
\[
a_1 = 5 + \frac{a_2}{2}
\]

**Best response for 2**:
\[
\frac{du_2}{da_2} = 10 - a_1 - 2a_2 = 0
\]
\[
2a_2 = 10 - a_1
\]
\[
a_2 = 5 - \frac{a_1}{2}
\]
\[
2a_1 = 10 + 5 - \frac{a_1}{2}
\]
\[
5 - \frac{a_1}{2} = 15
\]
\[
a_1 = 6
\]
\[
a_2 = 2
\]

**NE**: \((6, 2)\)