1 Review of Concepts

{dominant strategy} ⊆ {weakly dominant strategy} ⊆ {rationalizable strategy} ⊆ {best response strategy} = {undominated strategy}

Normal Form Games: If every player moves simultaneously and only once, a strategy is just an action $s_i = a_i$. If there are only two players and they have few actions the game is usually represented as a matrix. The first player picks the row, the second player picks the column and the payoffs are written as a pair of numbers in the respective cells.

Extensive Form Games: In more general games, players may move sequentially and repeatedly. In that case, a strategy is a complete contingent plan of actions. For example, in the Penny Auction a strategy is not just whether or not to bid a dollar initially, but a plan on how long to keep bidding additional dollars as a function of which competitors bid in the past, and possibly as a function of the amount of money in one's wallet (and one's belief about others' wallets).

Normal form and extensive form are, however, merely different representations for one game, rather than different categories of games. We will see in the course how games that would be naturally represented by their extensive form as a tree can also be represented in normal form, and vice versa.

2 Dominance

Definition: Dominance A strategy for a player is strictly dominated if and only if there is a single other strategy that yields a higher payoff, no matter what strategy all other players in the game pursue. A single strategy is dominant if it strictly dominates all other strategies. An outcome is consistent with iterated strict domination if it remains a possibility after all strictly dominated strategies are ruled out in turn.

Exercise 1: What (if any) strategies are strictly dominated in the following games? What outcomes are consistent with iterated strict domination?

\[
\begin{array}{c|cc|ccc|ccc|c}
& & & & & & & & & \\
\hline
& 1 & 2 & & & & & & & \\
\hline
1 & C & 3,3 & 1,4 & & & & & & \\
D & 4,1 & 2,2 & & & & & & & \\
\hline
\end{array}
\]

Exercise 2: Below is a simple (2x2) normal form game that could represent trade negotiations between Argentina and Brazil. Check for dominated strategies and then solve for all equilibria.

\[
\begin{array}{c|cc|ccc|ccc|c}
& & & & & & & & & \\
\hline
& F & P & & & & & & & \\
\hline
F & 3,3 & 1,4 & & & & & & & \\
P & 4,1 & 2,2 & & & & & & & \\
\hline
\end{array}
\]
Exercise 3: Consider an asymmetric alliance between China and Vietnam. Suppose China has a dominant strategy to build a strong military (rather than a weak one), but it would rather that Vietnam also build a strong military. Vietnam, meanwhile, wants to do the opposite of China’s action: it prefers to build a strong military if China builds a weak one, but prefers to build a weak military if China builds a strong one. Draw a 2x2 matrix for a game with payoffs consistent with these preferences. Check for dominated strategies and then solve for all equilibria.

Exercise 4: Kim-Jong Un has misbehaved again and the US and China need to decide whether to take action against him. Each one would like the other to take action. China doesn’t want to take action itself and the US would only take action if China does not. Overall, payoffs are given by:

<table>
<thead>
<tr>
<th></th>
<th>Action</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>3,3</td>
<td>2,4</td>
</tr>
<tr>
<td>No</td>
<td>4,1</td>
<td>1,2</td>
</tr>
</tbody>
</table>

Is this game dominance solvable? What’s the Nash equilibrium?

3 Elimination of Weakly Dominated Strategies

Technical/practical problem: The set of strategies that survive iterated deletion of weakly dominated strategies depends on the order in which strategies are deleted.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1,1</td>
<td>0,0</td>
</tr>
<tr>
<td>M</td>
<td>1,1</td>
<td>2,1</td>
</tr>
<tr>
<td>D</td>
<td>0,0</td>
<td>2,1</td>
</tr>
</tbody>
</table>

Example: The actions that survive iterative elimination of weakly dominated strategies can depend on the order in which the action are eliminated. For example, U can be eliminated since it is weakly dominated by M, and then L can be eliminated since it is weakly dominated by R. Now, Player 2 will choose action R, which will result in a payoff of (2; 1) for which ever action Player 1 selects. On the other hand, action D could have been eliminated first since it is weakly dominated by M, and then R could have been eliminated since it is weakly dominated by L. Now, the payoff is (1; 1) for which ever action Player 1 selects.

Exercise 5: Consider the following game in normal form, where player 1 chooses rows and player 2 chooses columns (in each cell, player 1s payoff is listed first and player 2s second). Does either player have dominated strategies? If so, what are they? Does either player have weakly dominated strategies? If so, what are they? What strategies are eliminated through iterative deletion of dominated choices? Is the game dominance solvable?

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5,6</td>
<td>7,5</td>
<td>1,5</td>
<td>0,8</td>
<td>2,4</td>
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<tr>
<td>B</td>
<td>3,4</td>
<td>8,6</td>
<td>2,3</td>
<td>1,0</td>
<td>11,2</td>
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<tr>
<td>C</td>
<td>2,3</td>
<td>12,3</td>
<td>6,2</td>
<td>3,4</td>
<td>9, 2</td>
</tr>
<tr>
<td>D</td>
<td>1,13</td>
<td>3,1</td>
<td>3,1</td>
<td>4,2</td>
<td>10,9</td>
</tr>
<tr>
<td>E</td>
<td>1,5</td>
<td>9,20</td>
<td>4,20</td>
<td>2,7</td>
<td>8, 4</td>
</tr>
</tbody>
</table>

4 Strategic Form Games

Exercise 6: In the Columbus-Ferdinand game below, suppose that if Columbus gets a high price then he is more likely to bring gold back to Ferdinand. How might this affect the outcome of the game and the strategies employed? Explain how you would change the characterization of the game in order to change the equilibrium outcome.
Exercise 7: Consider the extensive game below, representing a possible crisis in the Taiwan Strait. After some event (prior to the game) raises tensions, the United States first decides whether to send military personnel to the area, and then China must decide whether to instigate aggression or back down. The subgame-perfect (rollback) equilibrium outcome is for the United States to send its military and China to respond by backing down, yielding a payoff of (6,0). How might China be able to increase its payoff in equilibrium with each type of strategic move - commitment, threat, and promise? (If you like, you may add additional branch options at China’s choice nodes, explaining what they represent and choosing appropriate payoffs for each actor.)