

1 Review: Writing Strategic/Normal Form Games

**Definition: Rationality** Actors have a set of complete and transitive preferences, and make calculated, goal-oriented decisions over options with the aim of achieving their highest possible utility.

**Example** Building up a normal form trade game step-by-step:

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Outcomes</th>
<th>P1 Pref</th>
<th>Both Pref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Free Trade, Protect</td>
<td>FT</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>2: Free Trade, Protect</td>
<td>P</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Normal Form Games:** If every player moves simultaneously and only once, a strategy is just an action $s_i = a_i$. If there are only two players and they have few actions the game is usually represented as a matrix. The first player picks the row, the second player picks the column and the payoffs are written as a pair of numbers in the respective cells.

**Extensive Form Games:** In more general games, players may move sequentially and repeatedly. In that case, a strategy is a complete contingent plan of actions. For example, in the Penny Auction a strategy is not just whether or not to bid a dollar initially, but a plan on how long to keep bidding additional dollars as a function of which competitors bid in the past, and possibly as a function of the amount of money in ones wallet (and ones belief about others wallets).

Normal form and extensive form are, however, merely different representations for one game, rather than different categories of games. We will see in the course how games that would be naturally represented by their extensive form as a tree can also be represented in normal form, and vice versa.

2 Subgame Perfect Equilibria

**Definition: Subgame** A subset of an entire game that begins with a specific node and includes all successor nodes (and only successor nodes). The entire game is itself a subgame, as are any sub-parts of the game that can be split off according to this definition.
Definition: **Subgame-Perfect Equilibrium:** A Nash equilibrium in which the strategies involved prescribe a Nash equilibrium in every subgame (even those off the equilibrium path).

- Note: The set of subgame-perfect equilibria is a sub-set of the set of Nash equilibria - any SPE will always also be an NE. However, the reverse is not necessarily true: games may have NEs that are not SPEs.

- Note: Infinite games of perfect information without ties (the type of games for which SPE is a useful solution concept), subgame-perfect equilibrium is exactly equivalent to rollback equilibrium.

**Exercise 1** Consider the extensive form games below. In each, identify all subgames and all subgame-perfect equilibria. As an exercise, check if there are Nash equilibria that are not subgame-perfect (hint: yes, there are).

**Exercise 2** Consider the entry-deterrence game below (representing market competition). Firm 1 decides whether to enter a market or not (start producing a product), and firm 2 (which is already in the market) subsequently decides to fight it or accommodate. What is the subgame-perfect equilibrium of the game?

**Exercise 3** Consider again the entry-deterrence game from exercise 2. How would you represent this as a normal form game (changing the moves from sequential to simultaneous)?
Exercise 4 In each of the games below, identify the subgame perfect equilibrium, the equilibrium path, and the payoffs associated with that outcome. Be specific, clear, and complete.

a.

b.