Homework 5      PS 30      November 2018

Problems which you should be able to do easily

1. Consider the “Battle of the Sexes” game below.

\[
\begin{array}{cc}
1a & 2a \\
1b & 2b \\
\end{array}
\begin{array}{cccc}
2, 1 & 0, 0 \\
0, 0 & 1, 2 \\
\end{array}
\]

a. Find all Nash equilibria (pure strategy and mixed strategy) of this game.

b. Are any strategies in this game weakly or strongly dominated?

2. Consider the following game.

\[
\begin{array}{cccccc}
2a & 2b & 2c & 2d & 2e \\
1a & 63, -1 & 28, -1 & -2, 0 & -2, 45 & -3, 19 \\
1b & 32, 1 & 2, 2 & 2, 5 & 33, 0 & 2, 3 \\
1c & 54, 1 & 95, -1 & 0, 2 & 4, -1 & 0, 4 \\
1d & 1, -33 & -3, 43 & -1, 39 & 1, -12 & -1, 17 \\
1e & -22, 0 & 1, -13 & -1, 88 & -2, -57 & -3, 72 \\
\end{array}
\]

a. Find all pure strategy Nash equilibria of this game.

b. Make a prediction in this game by iteratively eliminating (strongly or weakly) dominated strategies.

3. [from Spring 2002 midterm] Mother can ask either Sister or Brother to do the dishes while she goes out shopping. If Mother asks Sister, she can either do it or not do it. If Mother asks Brother, he can either do it or not do it. Since Sister does a better job, Mother prefers Sister doing the dishes over Brother doing the dishes. However, Mother prefers Brother doing the dishes over them not being done.

Both Sister’s and Brother’s preferences are like this: the best thing is for the other person to do the dishes; the second best thing is for Mother to ask the other person and have the other person not do it (since then the other person will get blamed). The third best thing is to do the dishes, and the worst thing is to be asked to do the dishes but then not do it (since you will get in trouble).

a. Represent this as a strategic form game.

b. Find all (pure strategy) Nash equilibria.
More challenging problems

4. The simplest kind of game has two players, who each have two possible actions. We call these games “2 × 2 games.”

a. Write down a 2 × 2 game which has exactly one pure strategy Nash equilibrium and no mixed strategy Nash equilibrium. Solve for the equilibrium.

b. Write down a 2 × 2 game which has no pure strategy Nash equilibrium and exactly one mixed strategy Nash equilibrium. Solve for the equilibrium.

c. Write down a 2 × 2 game which has exactly three total (pure and mixed) Nash equilibria. Solve for the equilibria.

d. Write down a 2 × 2 game in which the total number of (pure and mixed) Nash equilibria is neither one nor three. Solve for the equilibria.

5. Say that country A and country I are at war. The two countries are separated by a system of rivers, as shown below.

```
A——b——c
   |
   d——e——f
   |
   g——h——l
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Country I sends a naval fleet with just enough supplies to reach A. The fleet must stop for the night at intersections (for example, if the fleet takes the path IhebA, it must stop the first night at h, the second at e, and the third at b). If unhindered, on the fourth day the fleet will reach A and destroy country A. Country A can send a fleet to prevent this. Country A’s fleet has enough supplies to visit three contiguous intersections, starting from A (for example Abcf). If it catches Country I’s fleet (that is, if both countries stop for the night at the same intersection), it destroys the fleet and wins the war. Model this as a strategic form game, assuming that the winner gets payoff 1 and the loser gets payoff -1. Iteratively eliminate weakly dominated strategies and make some sort of prediction.

6. [from Spring 2002 final] There are three people who each simultaneously choose the letter A or the letter B. If a person chooses a letter which no one else chooses, then he gets a payoff of 4; if a person chooses a letter which some other person chooses, then he gets a payoff of 0. The only exception is if everyone chooses the letter A, in which case everyone gets a payoff of 3.

a. Model this as a strategic form game and find all pure-strategy Nash equilibria.

b. There exists a mixed-strategy Nash equilibrium in which each person plays A with probability \( p \) and B with probability \( 1 - p \). Find \( p \).