1. Say that Townsville is deciding how many coal-fired energy plants to build to supply its energy needs. Some people are more environmentally oriented and thus prefer fewer plants, and some people think that the jobs and electricity that the plants provide are more important. Hence people differ on how many plants they feel are necessary. An opinion poll is taken asking each person how many plants she or he prefers. The results are that 14 percent of the population prefer 0 plants, 16 percent prefer 1 plant, 18 percent prefer 2 plants, 6 percent prefer 3 plants, 30 percent prefer 4 plants, and 16 percent prefer 5 plants.

da. Say that there are two candidates running for office, and the only relevant issue is how many plants to build. Each candidate takes a position on how many plants to build, and then each voter votes for the candidate which is closest to her own position or “ideal point.” For example, if candidate 1 is for 3 plants and candidate 2 is for 0 plants, a voter who prefers 2 plants will vote for candidate 1. If there is a “tie” (if two candidates are equally close to a voter’s ideal point), then half of the votes go to each candidate. For example, if candidate 1 is for 2 plants and candidate 2 is for 0 plants, then half of the people who prefer 1 vote for candidate 1 and half vote for candidate 2. Each candidate wants to maximize the total number of votes she gets.

Model this as a strategic form game (the candidates move simultaneously) as in the Downsian model. Find the pure strategy Nash equilibrium. Predict what positions the candidates will take and how many plants the town will build.

b. Now say that there are three candidates. Is there a pure strategy Nash equilibrium which is similar to what you found in part a.? (Don’t write down a three-person game! Just think about it.)

c. Now say that opinions shift. A new poll is taken, and it is found that 4 percent of the population prefer 0 plants, 10 percent prefer 1 plant, 78 percent prefer 2 plants, 2 percent prefer 3 plants, 2 percent prefer 4 plants, and 4 percent prefer 5 plants. Say there are two candidates. Predict what positions the candidates will take and how many plants the town will build.

d. Now again say that there are three candidates. Is there a pure strategy Nash equilibrium which is similar to what you found in part c.? (Don’t write down a three-person game! Just think about it.)
2. Say that Gotham City is deciding how many skate parks and dog walks to build in the city. A survey is done and it is found that 40 percent of the population are cranky taxpayers who dislike public expenditures and prefer 0 skate parks and 0 dog walks; 22 percent are hard core skate punks who prefer 6 skate parks and 0 dog walks; 30 percent are yuppie golden retriever owners who prefer 0 skate parks and 6 dog walks; and 8 percent are consensus-minded Buddhists who prefer 2 skate parks and 2 dog walks.

Say that there are two candidates running for office who take positions on both issues. As in the Downsian model, each voter votes for the candidate which is closest to her own position or “ideal point.” For example, if candidate 1 favors 1 skate park and 1 dog walk, and candidate 2 favors 4 skate parks and 2 dog walks, then candidate 1 gets 78 percent of the vote (the cranky taxpayers, the yuppies, and the Buddhists) and candidate 2 gets 22 percent (the skate punks). Each candidate wants to maximize the total number of votes she gets.

Math hint: in case you forgot, the distance between point \((a, b)\) and point \((c, d)\) is given by the Pythagorean Theorem: \(\sqrt{(a - c)^2 + (b - d)^2}\). For example, if candidate 1 favors 1 skate park and 1 dog walk, her position is \((1, 1)\). If candidate 2 favors 4 skate parks and 2 dog walks, her position is \((4, 2)\). The Buddhists have ideal point \((2, 2)\). Thus candidate 1 is distance \(\sqrt{(2 - 1)^2 + (2 - 1)^2} = \sqrt{2}\) away, and candidate 2 is distance \(\sqrt{(2 - 4)^2 + (2 - 2)^2} = \sqrt{4} = 2\) away. Since candidate 1 is closer, the Buddhists vote for candidate 1.

a. Let’s try to make a prediction in this game by eliminating weakly and strongly dominated strategies. First, simplify the game a lot by considering only the following strategies: \((0, 0)\), \((0, 3)\), \((1, 1)\), \((2, 2)\), \((3, 0)\), \((3, 3)\). Here \((3, 0)\) means for example 3 skate parks and 0 dog walks. Each of the two candidates thus has six possible strategies. Write this as a strategic form game and make a prediction by eliminating weakly and strongly dominated strategies.

b. Now say that most of the yuppies see Richard Gere movies and decide to become Buddhists (and take the Buddhist position of supporting 2 skate parks and 2 dog walks). Now there are 6 percent yuppies, 32 percent Buddhists, 40 percent cranky taxpayers, and 22 percent skate punks. Again, consider only the six strategies above and make a prediction by eliminating weakly and strongly dominated strategies. As the electorate becomes more “moderate,” do the candidates take more centrist positions?

3. The city council of Asbestosville wants to improve its image by bringing in the Palookaville Pirates, a minor league baseball franchise which is currently located in Palookaville. Asbestosville has built a brand new baseball field and is now trying to come up with other enticements for the Pirates, such as how much cash to give to the team. The city has already agreed to give the team $1 million, but some council members want to give the team more money. There are 11 council members. One member does not want to give the team any more money and prefers to give only $1 million total to the Pirates, one member prefers to give $2 million in total, one member prefers to give $3 million, one member prefers to give $4 million, and so forth; the eleventh council member wants to give the Pirates $11 million in total. As you can see, the median member of the council wants to give the Pirates $6 million.

a. The city council chairperson is a baseball fanatic and wants to give the Pirates $11 million in total (she is the eleventh council member). The chairperson controls the city agenda
and thus decides what proposal to bring to the council. When a proposal is brought to the council, all council members simply vote yes or no (the chairperson also votes). If the vote fails, then the policy remains at the status quo (giving $1 million total). Like in the Downsian model, a council member wants the final policy to be as close to her own “ideal point” as possible. What proposal will the chairperson make? How much money will the Pirates receive?

b. Now say that the mayor can veto the city council decision. The mayor’s ideal point is to give the Pirates a total of $2.6 million. If the council’s decision is farther away than the status quo from the mayor’s ideal point, then the mayor will veto the council’s decision and the status quo will be implemented. The sequence of decisions is like this: the council chairperson first makes a proposal, then the council members vote, and then the mayor can veto. Now what proposal will the chairperson make? How much money will the Pirates receive?

c. Now say that if the mayor vetoes the city council decision, the city council can override the veto with two thirds of the council vote (in this case, 8 of the 11 council members). If the council’s proposal is vetoed, then if the council overrides the veto, it can implement its original proposal. So now the sequence of decisions is like this: the council chairperson first makes a proposal, then the council members vote, and then the mayor can veto; if the mayor vetoes, then the council can override. Now what proposal will the chairperson make? How much money will the Pirates receive?