Homework 1  PS 204  Winter 2019

1. Say \( R \) is a preference relation defined on a set \( X \). We say that \( xPy \) if \( xRy \) and not \( yRx \); in other words, if \( xPy \), the person strongly prefers \( x \) over \( y \). We say that \( xIy \) if \( xRy \) and \( yRx \); in other words, if \( xIy \), the person is indifferent between \( x \) and \( y \).

a. Say \( x, y, z \in X \). Say that \( R \) is transitive. Say that \( xPy \) and \( yIz \). Show that \( xPz \).

b. Show that if \( R \) is transitive, then \( I \) is transitive.

c. Show that if \( R \) is transitive, then \( P \) is transitive.

2. Say that \( R \) is a preference relation defined on \( X \) and that \( R \) is complete and transitive. Show that \( P \) is “negatively transitive,” in other words, for all \( x, y \in X \), if \( xPy \), then for any \( z \in X \), either \( xPz \) or \( zPy \) or both.

3. Let \( X = \mathbb{R} \times \mathbb{R} \); in other words \( X \) is all ordered pairs of real numbers. Define the preference relation \( R \) on \( X \), where \((a, b)R(c, d)\) if either (1) \( a > c \) or (2) \( a = c \) and \( b \geq d \). For example, we have \((4, 2)R(3, 9)\) and we also have \((4, 2)R(4, 1)\). This preference relation is sometimes called a “lexicographic” preference because it is like alphabetizing words in the dictionary: order words according to the first letter, but if two words have the same first letter, order them according to the second letter.

a. Show that \( R \) is complete.

b. Show that \( R \) is transitive.

4. Let \( N = \{1, 2, 3, \ldots\} \) be the set of all natural numbers and let \( X = N \times N \). In other words, \( X \) is the set of ordered pairs \{(1, 1), (1, 2), (1, 3), \ldots, (2, 1), (2, 2), (2, 3) \ldots\}. We define the preference relation \( R \) on \( X \), where \((a, b)R(c, d)\) if \( \max \{a, b\} \geq \min \{c, d\} \). In other words, \((a, b)R(c, d)\) if the maximum of \( a \) and \( b \) is greater than the minimum of \( c \) and \( d \). For example, \((2, 5)R(100, 3)\).

a. Can you define a utility function \( u \) on \( X \) which represents the preference relation \( R \)? If so, find a utility function. If not, explain why not.

b. Now let \( X = \{(1, 1), (2, 2), (3, 3), \ldots\} \). Can you define a utility function \( u \) on \( X \) which represents the preference relation \( R \)? If so, find a utility function. If not, explain why not.
5. Let $X = \mathbb{R} \times \mathbb{R}$. We define the preference relation $R$ on $X$, where $(a, b)R(c, d)$ if $a \geq c$ or $b \geq d$.

a. Can you define a utility function $u$ on $X$ which represents the preference relation $R$? If so, find a utility function. If not, explain why not.

b. Now let $X = \{(1, 5), (2, 5), (3, 5), (4, 5), \ldots\}$. Can you define a utility function $u$ on $X$ which represents the preference relation $R$? If so, find a utility function. If not, explain why not.

c. Now let $X = \{(7, 6), (1, 5), (2, 5), (3, 5), (4, 5), \ldots\}$. Can you define a utility function $u$ on $X$ which represents the preference relation $R$? If so, find a utility function. If not, explain why not.

6. We say $\mathbb{R}^+$ is the set of nonnegative real numbers, in other words any $x \in \mathbb{R}$ such that $x \geq 0$. Let $X = \mathbb{R}^+ \times \mathbb{R}^+$. In other words, $X$ is the set of ordered pairs $(x, y)$, where $x \geq 0$ and $y \geq 0$. Say that a person likes both coffee and donuts, but because he is very superstitious, he really dislikes getting both 13 cups of coffee and 13 donuts. So we define the preference relation $R$ on $X$, where $(a, b)R(c, d)$ if $a + b \geq c + d$ or $(c, d) = (13, 13)$.

a. Can you define a utility function $u$ on $X$ which represents the preference relation $R$? If so, find a utility function. If not, explain why not.