1. a

\[ u_1(\xi, \eta, \zeta) = (140 - (\xi + \eta)) \eta \]

\[ u_2(\xi, \eta, \zeta) = (140 - (\eta + \zeta)) \xi - (\xi)^2 \]

1's best response?

Expanding to get:

\[ u_1(\hat{\xi}, \eta, \zeta) = 140 \eta - (\xi)^2 - \eta \xi \]

\[ \frac{du_1}{d\xi} = 140 - 2\xi - \eta \]

Set derivative equal to zero:

\[ 0 = 140 - 2\xi, -\eta \]

\[ 2\xi = 140, -\eta \]

\[ \xi = 70 - \frac{\eta}{2} \]

1's best response

2's best response?

\[ u_2(\hat{\xi}, \hat{\eta}, \zeta) = 140 \xi - \eta \zeta - (\eta)^2 - (\xi)^2 \]

\[ \frac{du_2}{d\eta} = 140 - 2\eta, -\xi \]

Set derivative equal to zero:

\[ 0 = 140 - 2\eta, -\xi \]

\[ 4\eta = 140, -\xi \]

\[ \eta = 35 - \frac{\xi}{4} \]

2's best response

The NE is where the best responses intersect.

\[ \xi = 70 - \frac{\eta}{2} \] 1's best response

\[ \eta = 35 - \frac{\xi}{4} \] 2's best response

Solve new equation:

\[ \xi = 70 - \frac{1}{2} \left(35 - \frac{\xi}{4}\right) \]

\[ 2\xi = 140 - 35 + \frac{\xi}{2} \]

\[ 2\xi = 105 \Rightarrow \xi = 60 \]

\[ \frac{7}{4} \xi = 60 \]

\[ \frac{7}{4} \frac{80}{2} - \frac{60}{2} = 20 \]

\[ s_0 (60, 20) \]

is the Nash equ.
16. Since \( u_1(g_1, q_2) = \left( 160 - (g_1 + g_2) \right) q_2 \),
we have \( u_1(0, q_2) = 0 \) for any \( q_2 \).

So I can always get utility \( 0 \) by playing \( q_1 = 0 \).

So \( q_1 = 150 \) (for example) is strongly dominated by \( q_1 = 0 \).

Because \( u_1(150, q_2) \) is negative for any \( q_2 \).

2a.

\[
\begin{align*}
u_1(a_1, a_2) &= 16 \frac{a_1}{a_1 + a_2} - a_1 \\
u_2(a_1, a_2) &= 16 \frac{a_2}{a_1 + a_2} - a_2
\end{align*}
\]

1's best response?

\[
\frac{\partial u_1}{\partial a_1} = 16 \frac{(a_1 + a_2) - a_1}{(a_1 + a_2)^2} - 1 = 0
\]

\[
16 a_2 = (a_1 + a_2)^2 \implies 4 \sqrt{a_1} = a_1 + a_2 \iff a_1 = 4 \sqrt{a_1} - a_2
\]

2's best response?

\[
\frac{\partial u_2}{\partial a_2} = 16 \frac{(a_1 + a_2) - a_2}{(a_1 + a_2)^2} - 1 = 0
\]

\[
16 a_1 = (a_1 + a_2)^2 \implies 4 \sqrt{a_2} = a_1 + a_2 \iff a_2 = 4 \sqrt{a_2} - a_1
\]

Nash eq? We know

\[
16 a_2 = (a_1 + a_2)^2
\]

and

\[
16 a_1 = (a_1 + a_2)^2
\]

Thus

\[
16 a_1 = 16 a_2 \implies a_1 = a_2.
\]

So

\[
\begin{align*}
16 a_1 &= (2a_1)^2 \\
16 a_1 &= 4 a_1^2
\end{align*}
\]

\[
\underline{4 = a_1} \quad \text{and} \quad \underline{4 = a_2}
\]

So \( \text{NE is} \ (4, 4) \).
Now \( U_1(a_1, a_2) = 16 \frac{a_1}{a_1 + a_2} - a_1 \),

\[ U_2(a_1, a_2) = 48 \frac{a_2}{a_1 + a_2} - a_2 \]

**Person 1's best response:**

16 \( a_2 = (a_1 + a_2)^2 \) \( \Rightarrow \) \( a_1 = 4, a_2 = a_2 - a_2 \)

**Person 2's best response?**

\[
\frac{\text{d}u_2}{\text{d}a_2} = 48 \frac{(a_1 + a_2) - a_2}{(a_1 + a_2)^2} - 1 = 0
\]

\[ 48 a_1 = (a_1 + a_2)^2 \Rightarrow a_2 = 4 \sqrt{3} a_1 - a_1 \]

To find NE, we have:

\[ 16 a_2 = (a_1 + a_2)^2 \]

\[ 48 a_1 = (a_1 + a_2)^2 \]

So \( 16 a_2 = 48 a_1 \)

\[ a_2 = 3 a_1 \]

So \( 48 a_1 = (a_1 + 3a_1)^2 \Rightarrow 48 a_1 = (4a_1)^2 \)

\[ 48 a_1 = 16 a_1^2 \]

\[ 3 a_1 = (a_1)^2 \Rightarrow a_1 = 3, a_2 = 3.3 = 9 \]

So NE is \((3, 9)\).

**Person 2 cares more about winning and hence spends more on warfare. Hence Person 2 weeks off.**
3. \( u(w, n) = nw \)
\[ n + w = 60 \Rightarrow n = 60 - w \]

0. \( u'(w) = (60 - w)w = 60w - w^2 \)
\[
0 = \frac{du}{dw} = 60 - 2w \Rightarrow 2w = 60 \Rightarrow w = 30 \Rightarrow n = 60 - 30 = 30 \\
\]

b. \( u_1(n, w_1) = nw_1 \)
\( u_2(n, w_2) = nw_2 \)
\( \bar{n} = n_1 + n_2 \)
\( n_1 + w_1 = 60 \Rightarrow n_1 = 60 - w_1 \)
\( n_2 + w_2 = 60 \Rightarrow n_2 = 60 - w_2 \)

\( u_1(w_1, w_2) = (n_1 + n_2)w_1 = (60 - w_1 + 60 - w_2)w_1 = (120 - w_1 - w_2)w_1 \)
\( u_2(w_1, w_2) = (120 - w_1 - w_2)w_2 \) similarly.

\[
\frac{d}{dw_1} = 120 - 2w_1 - w_2 = 0 \Rightarrow 120 - w_2 = 2w_1 \Rightarrow w_1 = 60 - \frac{w_2}{2} \\
\]
\[
\frac{d}{dw_2} = 120 - w_1 - 2w_2 = 0 \Rightarrow w_2 = 60 - \frac{w_1}{2} \]

so \( w_1 = 60 - \frac{1}{2}(60 - \frac{w_1}{2}) \)

\( 2w_1 = 120 - (60 - \frac{w_1}{2}) \)

\[ \frac{3w_1}{2} = 60 \Rightarrow w_1 = 40 \]

\[ w_2 = 60 - \frac{40}{2} = 40 \]

so NE is \( (40, 40) \)
\[ u_i(w_1, w_2, \ldots, w_m) = (b_0 - w_1 + b_0 - w_2 + \cdots + b_0 - w_m) w_i \]

\[ u_1(w_1, w_2, \ldots, w_m) = w_1 \]

\[ u_2(w_1, w_2, \ldots, w_m) = w_2 \]

\[ \frac{\partial u_1}{\partial w_1} = -w_1 + (b_0 - w_1 + b_0 - w_2 + \cdots + b_0 - w_m) = 0 \Rightarrow w_1 = b_0 - w_1 + b_0 - w_2 + \cdots + b_0 - w_m \]

\[ \frac{\partial u_2}{\partial w_2} = -w_2 + (b_0 - w_1 + b_0 - w_2 + \cdots + b_0 - w_m) = 0 \Rightarrow w_2 = b_0 - w_1 + b_0 - w_2 + \cdots + b_0 - w_m \]

Hence \( w_1 = w_2 = \cdots = w_m \).

So \( w_1 = (b_0 - w_1 + b_0 - w_2 + \cdots + b_0 - w_m) \)

\[ w_1 = b_0 - m w_1 \]

\[ (m+1)w_1 = b_0 m \]

\[ w_1 = \frac{b_0 m}{m+1} \]

\[ w_2 = w_3 = \cdots = \frac{b_0 m}{m+1} \]

So the NE is \( (b_0 \frac{m}{m+1}, b_0 \frac{m}{m+1}, \ldots, b_0 \frac{m}{m+1}) \)

\[ \text{with } w_1 = \frac{b_0 m}{m+1} \]

\[ v_1 = b_0 - b_0 \frac{m}{m+1} = b_0 \frac{(m+1) - bm}{m+1} = \frac{b_0}{m+1} \]

So as \( m \) grows large, each person makes less noise.

But the total amount of noise is \( \frac{b_0 m}{m+1} \).

Which approaches \( b_0 \) (the entire game) as \( m \) grows large.
If you spend $100 in Florida, either you win Florida
  (if your opponent spends less than $100, you tie everywhere)
  (if your opponent also spends $100 in Florida).

Either way, you get 30 delegates at least.

So there is no NE in which someone gets less than 30 delegates.

with 30 delegates total, in any NE, each person gets 30.

The only way this can happen is if they tie in all states
  or if some candidate wins Florida and one wins NC and GA.

In this case, for example

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Then this is not an NE (the person winning F can win another state and still)

If they tie in all states, for example

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Then this is not an NE because one candidate can win F by

So the only NE is if F cannot be correlated. (a)

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Now there is no NE because Romney can always put more money on all states than Gingrich.

Hence in any NE, Romney gets all 60 delegates and Gingrich 0.

However, Gingrich can always get at least some delegates by putting all his money in one state. (Romney can’t afford to put 60 in all three states).

Hence there is no pure strategy NE.