Homework 3  PS 172  February 2019

1. Say that persons 1 and 2 are thinking about watching the Hornets-Cavaliers game tonight. They are both Jeremy Lin fans but it is 50-50 whether Lin will be in the game because he is suffering from a sore ankle. Person 1 does not have a Tivo machine but person 2 does. So person 1 decides whether to watch the game live \( w \) or not \( n \), and person 2 decides whether to watch the game live \( w \) or Tivo the game \( t \). Payoffs are given as follows.

<table>
<thead>
<tr>
<th>( w )</th>
<th>( t )</th>
<th>( w )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>16,16</td>
<td>20,10</td>
<td>8,4</td>
</tr>
<tr>
<td>( n )</td>
<td>0,16</td>
<td>0,10</td>
<td>0,4</td>
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Lin in the game  Lin not in the game

Note that if Lin is not in the game, the best thing for person 1 is if person 2 Tivos it so he can see it later at person 2’s house; if person 2 does not Tivo it, then person 1 wants to watch the game. If Lin is in the game, person 1 wants to watch the game and also for person 2 to Tivo it so that he can go to person 2’s house later to relive the experience.

a. Say that neither person knows whether Lin will be in the game or not. Represent this as a strategic form game and find all Nash equilibria.

b. Say that the New York Post reports ahead of time whether or not Lin will be in the game or not, and both people read the Post. Represent this as a strategic form game and find all Nash equilibria.

c. Say that only person 1 reads the Post and hence only person 1 knows whether Lin will be in the game. Represent this as a strategic form game and find all Nash equilibria.

d. Say that only person 2 reads the Post and hence only person 2 knows whether Lin will be in the game. Represent this as a strategic form game and find all Nash equilibria.

e. Among a., b., c., and d., above, which is the best situation for person 1?

2. Say that persons 1 and 2 are thinking about whether to attend the protest tonight at Tiananmen Square. Each person can either revolt \( r \) or stay at home \( s \). But it is possible that the army will be called in to suppress the protest. The probability that the army will show up is 1/2 and the probability that the army will not is 1/2. Payoffs are given as follows.

<table>
<thead>
<tr>
<th>( r )</th>
<th>( s )</th>
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<tbody>
<tr>
<td>( r )</td>
<td>10,-8</td>
<td>-8,0</td>
<td>10,10</td>
</tr>
<tr>
<td>( s )</td>
<td>0,-8</td>
<td>0,0</td>
<td>0,10</td>
</tr>
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</table>

Army  No army

Note that no one wants to revolt alone; the only way a revolt is successful is if both people show up. Person 1 enjoys a successful revolt regardless of whether the army shows up or not, but Person 2 is skittish and likes a successful revolt only if the army is not there.
a. Say that neither person knows whether the army will show up or not. Represent this as a strategic form game and find all Nash equilibria.

b. Now say that both people know whether the army will show up or not. Represent this as a strategic form game and find all Nash equilibria.

c. Now say that only person 1 knows whether the army will show up or not. Represent this as a strategic form game and find all Nash equilibria.

d. Now say that only person 2 knows whether the army will show up or not. Represent this as a strategic form game and find all Nash equilibria.

e. Say that you are person 1 and you know whether the army shows up or not. Would you like for person 2 to have that information as well, or would you prefer that person 2 remain uninformed? (1 point)

f. Say that you are person 2 and person 1 knows whether the army shows up or not. Would you like to have that information also?

3. Say that there are two firms selling Frank Ocean t-shirts. If Ocean does not win a Grammy, then demand for t-shirts will be low: firm 1’s utility function is given by $u_1(\text{low}, q_1, q_2) = (48 - q_1 - q_2)q_1$ and firm 2’s utility function is given by $u_2(\text{low}, q_1, q_2) = (48 - q_1 - q_2)q_2$.

However, if Ocean wins a Grammy, then demand will be high: firm 1’s utility function is given by $u_1(\text{high}, q_1, q_2) = (60 - q_1 - q_2)q_1$ and firm 2’s utility function is given by $u_2(\text{high}, q_1, q_2) = (60 - q_1 - q_2)q_2$.

The probability that Ocean wins a Grammy is $1/2$ and the probability that he does not win is $1/2$.

a. Say that neither firm knows whether Ocean will win a Grammy. Model this as a game and find Nash equilibria.

b. Now say that both firms get inside information and both know whether Ocean will win a Grammy. Model this as a game and find Nash equilibria.

c. Now say that only firm 1 knows whether Ocean will win a Grammy; firm 2 remains ignorant. Model this as a game and find Nash equilibria.

d. If you are firm 1 and you know whether Ocean will win a Grammy, would you prefer that your competitor (firm 2) remain ignorant or would you prefer that firm 2 knows what you know?

e. If you are firm 2 and you do not know whether Ocean will win a Grammy, would you prefer that your competitor (firm 1) be ignorant also or would you prefer that firm 1 know whether Ocean will win?
4. Say that person 1 chooses a location $a_1$ and person 2 chooses a location $a_2$ on the beach, where $a_1$ and $a_2$ are both in the interval $[0, 1]$. The pier is at position $t = 0$ and the lifeguard station is at position $t = 1$. Person 1 likes to be close to person 2 but also the pier; hence her utility function is given by $u_1(a_1, a_2) = -(a_1 - a_2)^2 - (a_1 - 0)^2$. Person 2 likes to be close to person 1 and the lifeguard station. Hence person 2’s utility function is $u_2(a_1, a_2) = -(a_2 - a_1)^2 - (a_2 - 1)^2$.

a. Model this as a game and find Nash equilibria.

b. Now say that the pier is selling hot dogs. If person 1 has a hot dog craving, his utility function is given by $u_1(a_1, a_2, craving) = -(a_1 - a_2)^2 - 2(a_1 - 0)^2$. If person 1 does not have a hot dog craving, his utility function is given by $u_1(a_1, a_2, not) = -(a_1 - a_2)^2 - (a_1 - 0)^2$ as before. It is equally likely whether person 1 has a craving or not. Say that person 1 knows whether he has a craving but person 2 does not know. Person 2’s utility function is the same as before (person 2 never has a craving). Model this as a game and find Nash equilibria.

c. Now say that person 1 and person 2 both know whether person 1 has a craving. Model this as a game and find Nash equilibria.