This is a closed book exam. The only thing you can take into this exam is yourself and writing instruments. No calculators, computers, cell phones, or any electronic or computational devices are allowed. Everything you write should be your own work. Cases of academic dishonesty will be referred to the Dean of Students office, which has the power to suspend and expel students. Partial credit will be given: math mistakes will not jeopardize your grade. This exam has two parts. Each part is weighted equally (20 points each). Please show all steps of your work and explain what you are doing at each step. Correct answers alone are worth nothing without a clear and correct explanation of where the answers come from. Clarity and legibility are factors in the grade.

If you need to leave the room during the exam (to use the restroom for example), you need to sign your name on the restroom log before leaving. You can only leave the room once.

When the end of the exam is announced, please stop working immediately. The exams of people who continue working after the end of the exam is announced will have their scores penalized by 30 percent. When you hand in your exam, please write your name down on the log. Please write all answers on this exam—if you write on the reverse side of pages, please indicate this clearly. Good luck!
1. Person 1 and person 2 are playing a game, where each person can either cooperate (c) or defect (d). However, they are not sure what game they are playing. They could either be playing a Prisoners’ Dilemma (PD) or a Coordination Problem (CP). Both states are equally likely. Payoffs are given as follows.

\[
\begin{array}{cc|cc}
  & c & d & c & d \\
\hline
  c & 6,6 & 0,10 & 8,8 & -2,0 \\
  d & 10,0 & 2,2 & 0,-2 & 0,0 \\
\end{array}
\quad
\begin{array}{cc|cc}
  & c & d & c & d \\
\hline
  c & 6,6 & -2,0 & 8,8 & -1,1 \\
  d & 10,0 & -1,1 & 0,0 & 1,0 \\
\end{array}
\]

PD

CP

a. Say that both people are completely uninformed and do not know what game they are playing. Represent this as a strategic form game and find all Nash equilibria. (4 points)

b. Now say that both people are completely informed and know what game they are playing. Represent this as a strategic form game and find all Nash equilibria. (4 points)
Here are the payoffs again.

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>6,6</td>
<td>0,10</td>
</tr>
<tr>
<td>d</td>
<td>10,0</td>
<td>2,2</td>
</tr>
</tbody>
</table>

PD

c. Now say person 1 knows what game they are playing, but person 2 does not know what game they are playing. In other words, person 1 is informed but person 2 is uninformed. Represent this as a strategic form game and find all Nash equilibria. (4 points)

ME: (dd, cc), (dc, cc)
d. Now say again that both people do not know what game they are playing. Both are uninformed. Now say that the probability that the game is a Prisoners’ Dilemma is $p$ and the probability that the game is a Coordination Problem is $1 - p$. Write this as a strategic form game, in which the payoffs depend on $p$. (2 points)

\[
\begin{array}{c|cc}
\text{c} & \text{d} & \text{d} \\
\hline
\text{c} & 6, 6 & 6\rho + 8(-\rho), 6\rho + 8(1-\rho) \\
\text{d} & 0, 10 & -2(1-\rho), 10\rho \\
\end{array}
\]

\[
\begin{array}{c|cc}
\text{c} & \text{d} & \text{d} \\
\hline
\text{d} & 0, -2 & 2\rho, 2\rho \\
\end{array}
\]

\[\text{Simplify j:}\]

\[
\begin{array}{c|cc}
\text{c} & \text{d} & \text{d} \\
\hline
\text{c} & 8 - 2\rho, 8 - 2\rho & -2 + 2\rho, 10\rho \\
\text{d} & 10\rho, -2 + 2\rho & 2\rho, 2\rho \\
\end{array}
\]

e. The Nash equilibria of this game depend on the value of $p$. The higher $p$ is, the more likely the game is a Prisoners’ Dilemma and the more likely it is that people will play $d$. For example, if $p$ is close to 1, then both will play $d$ all the time. Circle the values of $p$ below for which there exists a Nash equilibrium in which people play $c$ at least sometime. Please explain your work. (2 points)

- $p = 0.1$
- $p = 0.2$
- $p = 0.3$
- $p = 0.4$
- $p = 0.5$
- $p = 0.6$
- $p = 0.7$
- $p = 0.8$
- $p = 0.9$
Here are the payoffs again.

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>6,6</td>
<td>0,10</td>
</tr>
<tr>
<td>d</td>
<td>10,0</td>
<td>2,2</td>
</tr>
</tbody>
</table>

PD

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>8,8</td>
<td>-2,0</td>
</tr>
<tr>
<td>d</td>
<td>0,-2</td>
<td>0,0</td>
</tr>
</tbody>
</table>

CP

f. Now say that person 1 knows what game they are playing but person 2 does not. Person 1 is informed but person 2 is not informed. Again, say that the probability that the game is a Prisoners’ Dilemma is $p$ and the probability that the game is a Coordination Problem is $1 - p$. Write this as a strategic form game, in which the payoffs depend on $p$. (2 points)

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>6p + 8(1-p)</td>
<td>0,10(1-p), 10p</td>
</tr>
<tr>
<td>d</td>
<td>6p, -2(1-p)</td>
<td>0, 10p</td>
</tr>
</tbody>
</table>

g. The Nash equilibria of this game depend on the value of $p$. The higher $p$ is, the more likely the game is a Prisoners’ Dilemma and the more likely it is that people will play $d$. For example, if $p$ is close to 1, then both will play $d$ all the time. Circle the values of $p$ below for which there exists a Nash equilibrium in which people play $c$ at least sometime. Please explain your work. (2 points)

- $p = 0.1$
- $p = 0.2$
- $p = 0.3$
- $p = 0.4$
- $p = 0.5$
- $p = 0.6$
- $p = 0.7$
- $p = 0.8$
- $p = 0.9$

\[ (d, d) \] is a NE if
\[ 8 + 2p > 10p \quad \text{and} \quad 8 - 8p > 2p \]
\[ 8 > 8p \quad \text{and} \quad 6 > 10p \]
\[ 1 > p \quad \text{and} \quad \frac{8}{5} > p. \]

So $\frac{8}{5} > p$. 

\[ (c, c, c) \quad \text{and} \quad \frac{8}{5} \geq p. \]

$\frac{8}{5} > p$. 

\[ (c, d, d) \quad \text{is a NE if} \]
\[ 4p - 2 > 2p \quad \text{and} \quad 2p > 8 - 8p \]
\[ 2p > 2 \quad \text{and} \quad 10p > 8 \]
\[ p > \frac{2}{3} \quad \text{and} \quad \frac{8}{5} > p. \]

So $p > \frac{2}{3}$. 

NE\{c,c,c\} because $2p > 2 + 2p$
2. Say that person 1 and person 2 are each deciding how many minutes to warm up for their audition for Hamilton. One wild card is that Broadway legend Audra McDonald is rumored to be in town and might actually show up for the audition, which would raise the stakes of the audition considerably. So either Audra will not be there (N) or she will be there (Y). Both states are equally likely.

Person 1 thrives on competition and the more that person 2 warms up, the more person 1 gains by warming up. Person 1’s utility functions are as follows:

\[ u_1(a_1, a_2, N) = (30 - a_1 + a_2) a_1 \]
\[ u_1(a_1, a_2, Y) = (50 - a_1 + a_2) a_1. \]

Person 2, on the other hand, dislikes competition and the more that person 1 warms up, the less person 2 gains by warming up. Person 2’s utility functions are as follows:

\[ u_2(a_1, a_2, N) = (30 - a_2 - a_1) a_2 \]
\[ u_2(a_1, a_2, Y) = (50 - a_2 - a_1) a_2. \]

a. Say that both people know whether Audra will be there or not. Find the Nash equilibrium of this game. (4 points)

\[ E_{u_1}^N = \frac{1}{2} (30 - a_1 + a_2^N) a_1^N + \frac{1}{2} (50 - a_1 + a_2^N) a_1^N \]
\[ E_{u_1}^Y = \frac{1}{2} (30 - a_1 - a_2^Y) a_1^Y + \frac{1}{2} (50 - a_1 - a_2^Y) a_1^Y \]

\[ \frac{dE_{u_1}}{da_1^N} = \frac{1}{2} (30 - 2a_1^N + a_2^N) = 0 \Rightarrow 2a_1^N = 30 + a_2^N \Rightarrow a_1^N = 15 + \frac{a_2^N}{2} \]
\[ \frac{dE_{u_1}}{da_1^Y} = \frac{1}{2} (50 - 2a_1^Y + a_2^Y) = 0 \Rightarrow 2a_1^Y = 50 + a_2^Y \Rightarrow a_1^Y = 25 + \frac{a_2^Y}{2} \]

\[ \frac{dE_{u_2}}{da_2^N} = \frac{1}{2} (30 - 2a_2^N - a_1^N) = 0 \Rightarrow 2a_2^N = 30 - a_1^N \Rightarrow a_2^N = 15 - \frac{a_1^N}{2} \]
\[ \frac{dE_{u_2}}{da_2^Y} = \frac{1}{2} (50 - 2a_2^Y - a_1^Y) = 0 \Rightarrow 2a_2^Y = 50 - a_1^Y \Rightarrow a_2^Y = 25 - \frac{a_1^Y}{2} \]

**ME:** \((Y, 30), (6, 10)\)

50 \ a_2^N = 15 + \frac{1}{2} (15 - \frac{a_1^N}{2})
4a_1^N = 60 + 30 - a_2^N
5a_1^N = 90
a_1^N = 18 \Rightarrow a_2^N = 15 - \frac{18}{2} = 6

5a_1^Y = 150
a_1^Y = 30 \Rightarrow a_2^Y = 25 - \frac{30}{2} = 10
Here are the utility functions again.

\[ u_1(a_1, a_2, N) = (30 - a_1 + a_2)a_1 \]
\[ u_1(a_1, a_2, Y) = (50 - a_1 + a_2)a_1 \]

\[ u_2(a_1, a_2, N) = (30 - a_2 - a_1)a_2 \]
\[ u_2(a_1, a_2, Y) = (50 - a_2 - a_1)a_2 \]

b. Say that neither person knows whether Audra will be there or not. Find the Nash equilibrium of this game. (4 points)

\[ E_{u_1} = \frac{1}{2} \left( 30 - a_1 + a_2 \right) a_1 + \frac{1}{2} \left( 50 - a_1 + a_2 \right) a_2 \]
\[ E_{u_2} = \frac{1}{2} \left( 30 - a_2 - a_1 \right) a_2 + \frac{1}{2} \left( 50 - a_2 - a_1 \right) a_2 \]

\[ \frac{dE_{u_1}}{da_1} = \frac{1}{2} \left( 30 - 2a_1 + a_2 \right) + \frac{1}{2} \left( 50 - 2a_1 + a_2 \right) = 0 \]
\[ 80 - 4a_1 + 2a_2 = 0 \Rightarrow a_1 = 20 + \frac{a_2}{2} \]

\[ \frac{dE_{u_2}}{da_2} = \frac{1}{2} \left( 30 - 2a_2 - a_1 \right) + \frac{1}{2} \left( 50 - 2a_2 - a_1 \right) = 0 \]
\[ 80 - 4a_2 - 2a_1 = 0 \Rightarrow a_2 = 20 - \frac{a_1}{2} \]

So \[ a_1 = 20 + \frac{1}{2} \left( 20 - \frac{a_2}{2} \right) \]
\[ 4a_1 = 80 + 40 - a_1 \]
\[ 5a_1 = 120 \]
\[ a_1 = 24 \Rightarrow a_2 = 20 - \frac{24}{2} = 8 \]

\[ \text{NE: } (24, 24), \ (8, 8) \]
Here are the utility functions again.

\[ u_1(a_1, a_2, N) = (30 - a_1 + a_2)a_1 \]
\[ u_1(a_1, a_2, Y) = (50 - a_1 + a_2)a_1 \]
\[ u_2(a_1, a_2, N) = (30 - a_2 - a_1)a_2 \]
\[ u_2(a_1, a_2, Y) = (50 - a_2 - a_1)a_2 \]

c. Say that person 1 knows whether Audra will be there or not, but person 2 does not know. Find the Nash equilibrium of this game. (4 points)

From part a, we have
\[ a_1^h = 15 + \frac{a_2}{2} \quad a_1^y = 25 + \frac{a_2}{2} \]

\[ EU_2 = \frac{1}{2} (30 - a_2 - a_1^h) a_1 + \frac{1}{2} (50 - a_2 - a_1^y) a_2 \]

\[ \frac{dEU_2}{da_2} = \frac{1}{2} (30 - 2a_2 - a_1^h) + \frac{1}{2} (50 - 2a_2 - a_1^y) = 0 \]
\[ 80 - 4a_2 - a_1^h - a_1^y = 0 \]
\[ 80 - a_1^h - a_1^y = 4a_2 \]
\[ 20 - \frac{4}{4} (a_1^h + a_1^y) = a_2 \]

So
\[ a_2 = 20 - \frac{4}{4} \left( 15 + \frac{a_2}{2} + 25 + \frac{a_2}{2} \right) \]
\[ 4a_2 = 80 - (40 + a_2) \]
\[ 5a_2 = 40 \]
\[ a_2 = 8 \quad a_1^h = 15 + \frac{8}{2} = 19 \quad a_1^y = 25 + \frac{8}{2} = 32 \]

NE: \((19, 2, 9, 8, 8)\)
Here are the utility functions again.

\[ u_1(a_1, a_2, N) = (30 - a_1 + a_2)a_1 \]
\[ u_1(a_1, a_2, Y) = (50 - a_1 + a_2)a_1 \]

\[ u_2(a_1, a_2, N) = (30 - a_2 - a_1)a_2 \]
\[ u_2(a_1, a_2, Y) = (50 - a_2 - a_1)a_2 \]

d. Say that person 2 knows whether Audra will be there or not, but person 1 does not know. Find the Nash equilibrium of this game. (4 points)

From \( \pi A \) \( b \), \( a_2^n = 15 - \frac{a_1}{2} \) \( a_2^y = 25 - \frac{a_1}{2} \)

\[ E u_1 = \frac{1}{2} \left( 30 - a_1 + a_2^n \right) a_1 + \frac{1}{2} \left( 50 - a_1 + a_2^y \right) a_1 \]

\[ \frac{d E u_1}{d a_1} = \frac{1}{2} \left( 30 - 2a_1 + a_2^n \right) + \frac{1}{2} \left( 50 - 2a_1 + a_2^y \right) = 0 \]
\[ 80 - 4a_1 + a_2^n + a_2^y = 0 \]
\[ 4a_1 = 80 + a_2^n + a_2^y \]
\[ a_1 = 20 + \frac{1}{4} (a_2^n + a_2^y) \]

So \( a_1 = 20 + \frac{1}{4} (15 - \frac{a_1}{2} + 25 - \frac{a_1}{2}) \)
\[ 4a_1 = 80 + 15 - \frac{a_1}{2} + 25 - \frac{a_1}{2} \]
\[ 5a_1 = 120 \]
\[ a_1 = 24 \]
\[ a_2^n = 15 - \frac{24}{2} = 3 \]
\[ a_2^y = 25 - \frac{24}{2} = 13 \]

\( \text{NE} : (24, 24, 3, 13) \)

e. Of all of the four scenarios above, which is the best for person 2? (4 points)

\[ E u_2 = \frac{1}{2} \left( 30 - a_2 - a_1 \right) a_2 + \frac{1}{2} \left( 50 - a_2 - a_1 \right) a_2 \]

\( a : (18, 32, 6, 10) \) \( E u_2 = \frac{1}{2} (30 - 24) 6 + \frac{1}{2} (50 - 24) 10 = \frac{1}{2} 36 + \frac{1}{2} 100 = 68 \)

\( b : (2, 24, 6, 8) \) \( E u_2 = \frac{1}{2} (30 - 27) 8 + \frac{1}{2} (50 - 27) 8 = \frac{1}{2} (16) 8 = 64 \)

\( c : (18, 29, 8, 8) \) \( E u_2 = \frac{1}{2} (30 - 27) 8 + \frac{1}{2} (50 - 37) 8 = \frac{1}{2} (13) 8 = 64 \)

\( d : (2, 29, 3, 19) \) \( E u_2 = \frac{1}{2} (30 - 27) 3 + \frac{1}{2} (50 - 37) 13 = \frac{1}{2} 9 + \frac{1}{2} 169 = 89 \)

So \( d \) is best for person 2.