1. Say country 1 and country 2 are in a conflict and each must decide whether to use a military option \(m\) or the diplomatic option \(d\). Country 1 has been trying to develop laser rifles, which would give it a very large military advantage. Country 2 has been trying to place a spy in Country 1’s weapons labs. Therefore there is uncertainty about whether Country 1 has laser rifles \(l\) or does not have laser rifles \(n\), and there is uncertainty about whether Country 2 has successfully placed a spy \(s\) or not \(n\). So there are four states of the world: 

\[
\begin{array}{cc}
ls & ln \\
ns & nn
\end{array}
\]

For example, \(ns\) is the state in which Country 1 does not have laser rifles and Country 2 has successfully placed a spy. Each state is equally likely.

If Country 1 has successfully developed laser rifles (states \(ls\) and \(ln\)), then payoffs are as follows.

\[
\begin{array}{cc}
m & d \\
m & 4, -8 & 8, -20 \\
d & -20, -4 & 0, 0
\end{array}
\]

If Country 1 has not developed laser rifles (states \(ns\) and \(nn\)), then payoffs are as follows.

Note that if the other country chooses the military option and you don’t, you suffer greatly.

\[
\begin{array}{cc}
m & d \\
m & -8, -8 & -4, -20 \\
d & -20, -4 & 0, 0
\end{array}
\]

a. Consider the “standard” scenario. Country 1 of course knows whether it has laser rifles or not, but does not know whether Country 2 has placed a spy or not. If Country 2 has placed a spy, then it knows whether Country 1 has laser rifles or not, but if Country 2 has not placed a spy, then it does not know. Model this as a game with incomplete information and find all Bayesian Nash equilibria. (Note that when Country 1 has laser rifles, then \(m\) dominates \(d\). Note also that when Country 2 knows that Country 1 has laser rifles, then Country 2 will therefore surely play \(m\). This allows you to simplify matters a lot: you only have to consider (at most) two possible strategies for Country 1 and (at most) four possible strategies for Country 2.)

b. Now consider the “self-busted spy” scenario. As in the standard scenario, if Country 2 has placed a spy, then it knows whether Country 1 has laser rifles or not, and if Country 2 has not placed a spy, then it does not know. Country 2’s information partition is the same as before.

However, say that when Country 2’s spy reports back that Country 1 has not developed laser rifles, Country 2 reveals the existence of its spy to Country 1. In other words, when Country 1 has not developed laser rifles, it knows whether or not Country 2 has placed a spy. Model this as a game with incomplete information and find all Bayesian Nash equilibria. (Again, note that when Country 1 has laser rifles, then \(m\) dominates \(d\). Note also that when Country 2 knows that Country 1 has laser rifles, then Country 2 will therefore surely play
m. Hence you only have to consider (at most) four possible strategies for Country 1 and (at most) four possible strategies for Country 2.)

c. Now consider the “inspection regime” scenario. As in the standard scenario, Country 1 does not know whether Country 2 has placed a spy or not; all Country 1 knows is whether it has laser rifles or not. However, now Country 1 fully discloses its weapons program to Country 2. Now Country 2 knows whether or not Country 1 has laser rifles regardless of whether it placed a spy or not. Country 2 knows everything. Model this as a game with incomplete information and find all Bayesian Nash equilibria. (Again, note that when Country 1 has laser rifles, then \( m \) dominates \( d \). Note also that when Country 2 knows that Country 1 has laser rifles, then Country 2 will therefore surely play \( m \). Hence you only have to consider (at most) two possible strategies for Country 1 and (at most) four possible strategies for Country 2.)

d. Which of these three scenarios is most favorable to peace and why?

2. Alfonso and Beatrice are home for the holidays and are eating some holiday cookies. They decide to play a game. They take turns eating cookies. In each turn, a person can either take 1, 2, or 3 cookies. However, a person cannot take more cookies than what her opponent took last time. In other words, if your opponent just took 2 cookies, you can only take 1 or 2 cookies (you cannot take 3). Whoever eats the last cookie wins. Alfonso goes first.

a. Say that Alfonso and Beatrice start with a plate of 6 cookies. Model this as an extensive form game. Write payoffs as (Alfonso, Beatrice).

b. Find a subgame perfect Nash equilibrium of this game by writing arrows in the tree you wrote above (i.e. you don’t have to write down the SPNE in words). Please make your arrows nice and clear. If there is more than one SPNE, just write down one of them; you don’t have to find all of them.

c. Now say that they start with \( x \) cookies, where \( x \) goes from 1 to 20, as shown in the table below. For each value of \( x \), find out which person wins the game in an SPNE and write it in the table below. Remember that Alfonso always goes first. For example, when \( x = 1 \), there is only one cookie at the start, and Alfonso obviously wins by taking one cookie right at the start. So the table entry when \( x = 1 \) is already filled in for you as an example.
3. Say that two people each decide whether to go out on the town $t$ or stay home $s$. If you stay home, you get a payoff of 0. If you go out on the town alone, then you are unhappy and get a payoff of $-4$. If you both decide to go out, then the two of you get picked up by a mystery cab which can take the two of you to four possible places: karaoke $k$, blues bar $b$, disco $d$, or cigar lounge $c$. Each location is equally likely (each takes place with probability $1/4$). The payoffs from each look like this:

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$s$</th>
<th>$t$</th>
<th>$s$</th>
<th>$t$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$-12$</td>
<td>$28$</td>
<td>$-4$</td>
<td>$0$</td>
<td>$t$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$s$</td>
<td>$0$</td>
<td>$-4$</td>
<td>$0$</td>
<td>$0$</td>
<td>$s$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$s$</th>
<th>$t$</th>
<th>$s$</th>
<th>$t$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$8$</td>
<td>$-8$</td>
<td>$-4$</td>
<td>$0$</td>
<td>$t$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$s$</td>
<td>$0$</td>
<td>$-4$</td>
<td>$0$</td>
<td>$0$</td>
<td>$s$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$s$</th>
<th>$t$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$16$</td>
<td>$-8$</td>
<td>$-4$</td>
<td>$0$</td>
</tr>
<tr>
<td>$s$</td>
<td>$0$</td>
<td>$-4$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$s$</th>
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</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$-4$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$s$</td>
<td>$-4$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Karaoke</th>
<th>Blues</th>
<th>Disco</th>
<th>Cigar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$-12$</td>
<td>$8$</td>
<td>$16$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$s$</td>
<td>$28$</td>
<td>$-8$</td>
<td>$-8$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

a. Say that neither person knows where the mystery cab will go to before deciding whether to go out or not. Represent this as a game and find all Nash equilibria.

b. Now say that person 1 doesn’t know where the mystery cab will go to, but person 2 knows ahead of time whether the mystery cab will go to karaoke or not (person 2 can’t tell whether the mystery cab will go to the blues bar, disco, or cigar lounge, but she can tell whether it will go to karaoke or not). Represent this as a game and find all Nash equilibria.
c. Now consider all possible knowledge partitions for person 1 and for person 2. Draw the partitions for person 1 and for person 2 which yield a Bayesian Nash equilibrium which is best for person 1. Draw the partitions for person 1 and for person 2 which yield a Bayesian Nash equilibrium which is best for person 2.

4. Consider the following two-person game, in which Nature moves first.

Person 1 is applying for a sales job, and Person 2 is the company deciding whether to hire person 1 or not. First, Nature chooses whether Person 1 has research skills or sales skills—each has probability $1/2$. If person 1 has research skills, she decides whether to get an MBA or not. Similarly, if person 1 has sales skills, she decides whether to get an MBA or not. Finally, Person 2 decides whether to hire $h$ or not $n$. Person 1 knows what kind of skills she herself has, but Person 2 doesn’t know Person 1’s skills. Person 2 can tell whether Person 1 got an MBA or not, as that information is on her resume.

If Person 1 does not get an MBA and is not hired, then Person 1 gets 0. If Person 1 gets an MBA and is not hired, he gets $-8$ because getting an MBA costs 8. If Person 2 does not hire, Person 2 gets a payoff of 0.

Now if Person 1 has sales skills, she anticipates getting large sales commissions and thus if she is hired (without having an MBA) she gets a payoff of 10. If Person 1 has research skills, she expects small sales commissions, and thus if she is hired (without having an MBA) she gets a payoff of 6. As before, getting an MBA costs 8.

Person 2 gets a payoff of 8 if it hires someone with sales skills (it is a sales job, after all). If it hires someone with the wrong skills, it gets a payoff of $-10$.

This model tries to explain why people might get costly degrees that do not make them more useful to their employer (Person 2 doesn’t care whether Person 1 has an MBA or not).
This is the “signalling” model of education (google “Michael Spence Nobel”). The question is what an employer learns when she sees an MBA on an applicant’s resume. This depends on which people (what kind of skills) get an MBA. But which people choose to get an MBA depends on what an employer learns when seeing an MBA on a resume.

Find the Perfect Bayesian Nash equilibria of this game. Note that both person 1 and person 2 have four strategies.