ADVERSE SELECTION: THE ROTHSCCHILD–STIGLITZ MODEL

We have examined Akerlof's model of the market for lemons, which illuminates the destructive potential of asymmetric information in a transactional market. We have also examined the underlying structure of risk aversion, and analyzed insurance contracts to determine the expected amount of utility they will yield. Economists Michael Rothschild and Joseph Stiglitz developed a model that puts these two insights together (Rothschild and Stiglitz 1976). Our goal in this chapter is to describe this model and present some conclusions about how insurance markets deal with adverse selection.

9.1 The \( I_H-I_S \) space

We start by developing a framework to display multiple insurance contracts simultaneously and intuitively. Consider Figure 9.1(a), an example of the income–utility diagram we developed in Chapter 7. Using this diagram, it is fairly easy to evaluate a single insurance contract to see if it is utility-enhancing when compared with the uninsured state. But once we try to examine more than one insurance contract at a time, the graph quickly becomes illegible.

One way around the problem of displaying multiple contracts is to plot the same insurance contracts on different axes. We let each axis plot income in one state of the world, healthy on the horizontal axis and sick on the vertical axis. These quantities are both plotted along the income axis in Figure 9.1(a), but in Figure 9.1(b) the \( I_H \) and \( I_S \) are set perpendicular to one another. The chief insight here is that insurance contracts transfer income from the healthy state of the world to the unhealthy. By plotting \( I_H \) and \( I_S \) in the same space, we can better visualize the transfers that different contracts imply.

As can be seen in Figure 9.1(a), the individual receives an income of \( I_H \) if she remains healthy and an income of \( I_S \) if she becomes sick, assuming no insurance. We call the point \( E = (I_H, I_S) \) the individual's endowment. In Figure 9.1(b), the endowment \( E \) is plotted in the \( I_H-I_S \) space. The endowment point \( E \) reflects all the information about the individual's income in the healthy and sick states.

The same graph can also be used to represent income in various states of the world for an individual who is insured. Let \( H_C \) and \( S_C \) represent income in the healthy and sick states for the same individual with partial insurance contract \( C \). Figure 9.2(a) shows this partial insurance contract in the familiar income–utility space. Figure 9.2(b) more concisely shows this contract as a single point \( I \) and contains the same information about the insured individual's possible income levels under partial insurance.

9.2 Indifference curves in \( I_H-I_S \) space

Our next job is to introduce utility into the \( I_H-I_S \) space. This is easy in the income–utility space from Chapter 7, since utility is plotted on the vertical axis in that space. In the \( I_H-I_S \) space, though, we must introduce indifference curves to compare potential insurance contracts.

Figure 9.3 shows the endowment point and four potential insurance contracts \( C_1 \) to \( C_4 \). Even without knowing much about the individual's preferences, we make some evaluations about the relative utility from these contracts. Consider, for instance, the difference between contracts \( C_3 \) and \( C_4 \). Both contracts result in the same sick-state income \( a_1 \), but contract \( C_4 \) gives a higher healthy-state income than does \( C_3 \). As long as the individual prefers more income to less, she always prefers \( C_4 \) to \( C_3 \). For similar reasons, the individual always prefers \( C_1 \) to \( C_4 \). Still, without further information about the individual's...
preferences, we cannot determine whether she prefers \( C_1 \) to \( C_4 \), or how any of these contracts compare with no insurance \( E \).

We need indifference curves, but it is not immediately obvious how to translate the utility curve from the income–utility diagram into indifference curves in \( l_H - l_I \) space.

These indifference curves will have two basic properties. First, they are downward-sloping. This follows from the tradeoff between \( l_H \) and \( l_I \) in an insurance contract. An individual will be willing to sacrifice income in one state only if she is compensated with more income in the other state. For instance, it is possible for someone who likes income to have an indifference curve running through \( C_1 \) and \( C_4 \), but never possible to have an indifference curve running between \( C_2 \) and \( C_3 \). This fact can also be proved mathematically using differential equations (see Exercise 17).

In addition to being downward-sloping, indifference curves are also convex whenever consumers are risk-averse. In other words, they are steeply downward-sloping at low levels of \( l_H \) but flatten out at higher levels of \( l_H \). Convex indifference curves imply that \( l_H \) and \( l_I \) are substitutes, but not perfect ones. The individual is willing to trade off some \( l_H \) for \( l_I \) and vice versa but prefers to maintain at least moderate levels of both. This is a natural consequence of risk aversion (see Exercise 17 for a mathematical proof).

Figure 9.4 displays two downward-sloping and convex indifference curves in the \( l_H - l_I \) space. Indifference curve \( U_1 \) shows that the individual is indifferent between points \( C_1, C_3 \), and \( E \).

9.3 The full-insurance line

The full-insurance line is an important landmark in \( l_H - l_I \) space. By definition, a full-insurance contract is one that achieves state independence, \( l_I = l_H \). A fully insured individual is guaranteed the same income whether she is sick or healthy. The 45° line defined by the equation \( l_H = l_I \) is pictured in Figure 9.5 and covers the range of all full-insurance contracts. Any insurance contract not on this line cannot be a full-insurance contract, because \( l_H \neq l_I \).

In Figure 9.5, three insurance contracts are displayed. All fit the definition of full insurance, because \( l_I = l_H \) in every case. But the three contracts offer different levels of utility. \( C_1 \) is preferable to \( C_2 \), which is preferable to \( C_3 \). In fact, the individual's indifference curve indicates that she is indifferent between \( C_2 \) and \( E \), and prefers \( E \) to \( C_3 \). In Section 9.4, we show that these preferences are driven by the fact that \( C_2 \) is more unfair than \( C_4 \), and \( C_3 \) is more unfair than \( C_1 \).

9.4 The zero-profit line

Just as there is a line characterizing all full-insurance contracts in the \( l_H - l_I \) space, there is also a line characterizing all actuarily fair contracts.

The actuarial fairness of a contract depends on the probability \( p \), that the individual falls ill. If the individual is very likely to be sick, a certain set of contracts with high premiums and low payouts will be actuarially fair. If the individual is very likely to be healthy, a different set of contracts with high premiums and low payouts will be fair.

Suppose an individual with probability \( p \) starts with an income at endowment point \( E = (H_0, S_0) \). Her income without insurance is \( H_0 \) if she is healthy, and \( S_0 \) if she falls ill. Consider an arbitrary insurance contract that moves the individual to a point...
(H_E, S_C). This contract is actuarially fair if the insurer's expected profit from this contract is zero (or equivalently if the customer's change in expected income is zero). The insurer gains \( H_E - H_C \) if the individual stays healthy, but loses \( S_C - S_E \) if he becomes sick. In expected value, the insurer makes zero profit if

\[
p(S_C - S_E) = (1 - p)(H_E - H_C)
\]

This expresses the same actuarial-fairness condition from Chapter 7, except that \( H_E - H_C \) is the premium \( r \) and \( S_C - S_E \) is payout net premium \( q - r \).

Rearranging equation (9.1) yields

\[
S_C = \frac{1 - p}{p} (H_E - H_C) + S_E
\]

We have been considering an arbitrary contract \( C = (H_C, S_C) \), but this equation plots out the set of contracts that have zero profit for the insurer. If we replace the contract with an arbitrary actuarially fair contract \( (H_C, S_C) \), the condition would still hold.

\[
I = \frac{1 - p}{p} (H_E - I) + S_E
\]

This equation defines a zero-profit line in \( I - I \) space with slope \( \frac{1 - p}{p} \). The line will always run through the endowment point \( E \), which confirms our intuition that the endowment point uninsurance is actuarially fair. Importantly, the zero-profit line's slope changes with \( p \); a more sickly person with higher \( p \) has a flatter slope than a less sickly person with a lower \( p \). By contrast, the slope of the full-insurance line does not change with \( p \).

Figure 9.6 shows an example of a zero-profit line in \( I - I \) space. In this figure, three actuarially fair insurance contracts are displayed, alongside the endowment contract \( E \). The other contracts are all preferable to \( E \) — any contract with fair insurance, whether partial or full, will be preferred to the endowment point.

Also consider the insurance company's preferences. Just as consumers generally prefer actuarially fair insurance to unfair insurance, insurance companies prefer the opposite. They like the profitable, actuarially unfair contracts better than zero-profit actuarially fair contracts. In the \( I - I \) space, contracts in the zone southwest of the zero-profit line make positive profits, and contracts in the northeast zone make negative profits. Figure 9.7 demonstrates why this is so.

Consider contracts \( C_1, C_2, \) and \( C_3, C_4 \) on the zero-profit line and hence makes zero profit. \( C_1 \) charges the same premium \( H_E - H_C \) as \( C_2 \) but pays out less, since \( S_2 > S_1 \).

This is true for all contracts in the southwest zone; one can always find a contract on the zero-profit line that requires the same premium but offers a higher payout. Thus, contracts in this region always make positive profits for the insurer. Similarly, contracts in the northeast region like \( C_3 \) always make negative profits for the insurer, because these contracts pay out more for the same premium.

### 9.5 The feasible contract wedge

Our next task is to determine where in the \( I - I \) space insurance companies and customers would be happy to meet and do business. The insurance market unravels if there are no contracts that mutually benefit both companies and customers.

So far, we have discussed the full-insurance line, the zero-profit line, and indifference curves — for now, we focus on the indifference curve passing through the endowment point. These lines and curves divide up the \( I - I \) space into four regions, labeled \( R_1, R_2, R_3, \) and \( F \) in Figure 9.8. In which of these regions, if any, can contracts be offered and accepted?
risk-averse individuals would never seek. Thus, the market will produce no contracts in region $R_i$.

- Region $R_c$ contains contracts that all lie to the southwest of the indifference curve passing through $E$. Therefore, the customers always prefer uninsured to any contract offered in this region. These contracts cost so much and offer so little in return that uninsured is preferable. No successful contract can exist here.

- Region $R_a$ contains only unprofitable insurance contracts as it lies entirely northeast of the zero-profit line (see Figure 9.7). No insurance company would be willing to offer insurance contracts here.

- This leaves region $R$. It is labeled the feasible contract wedge because it is the only region where contracts can exist. All the contracts in the feasible contract wedge are at least weakly preferable to $E$ and represent positive (or at least zero) profits. Note that all points on the edge of the wedge are feasible as well.

### 9.6 Finding an equilibrium

Identifying the set of feasible contracts is a step toward determining whether contracts will actually sell in this market. Next, we need to define an equilibrium in this market. A market equilibrium satisfies three conditions: consumers maximize utility, firms maximize profits, and no new firm can enter the market without incurring negative profits. We formalize these conditions below.

**Definition 9.1**

A set of contracts is in equilibrium if:

1. all individuals select the contract in the equilibrium set that offers the most utility;
2. no contract in the set earns negative profits for the firm offering it;
3. there exists no contract or set of contracts outside the equilibrium set that, if offered, would attract customers and earn at least zero profit.

Any set of insurance contracts that fulfills these three rules is a valid equilibrium. At this point, it may be unclear to readers why we need to distinguish between the second and third equilibrium conditions. In the typical model of consumer demand, these conditions are combined into a single one: in equilibrium, firms make zero profit. As we will see, this distinction becomes important when we introduce asymmetric information.

Let us consider the case with no information asymmetry and a homogeneous set of consumers, all alike with probability of illness $p$ and endowment point $E$. Any potential equilibrium must lie in the feasible contract wedge; consider a contract $\alpha$ in that region (Figure 9.9). Is $\alpha$ an equilibrium?

All consumers would buy contract $\alpha$, because it lies on a higher indifference curve than $E$. Thus, $\alpha$ satisfies the first equilibrium condition.

Furthermore, $\alpha$ lies southwest of the zero-profit line, so it generates positive profits for the insurance company. Thus, $\alpha$ satisfies the second equilibrium condition.

However, $\alpha$ does not meet the third equilibrium condition: there is an opening for a different insurance company to offer a profitable contract that induces all the customers to switch from $\alpha$. Such a contract would necessarily be located between $U_a$ and the zero-profit line, say at $\beta$ in Figure 9.9. If a second insurance company were to offer $\beta$, the consumers would switch from $\alpha$ to $\beta$. Hence, $\alpha$ is not in the stable equilibrium set.

The same logic reveals that $\beta$ cannot be an equilibrium contract either. As long as the proposed contract lies in the interior of the feasible contract wedge, an enterprising insurance company can offer a different contract that makes positive profits.

Only when an actuarially fair, full-insurance contract is offered ($\Omega$) are the equilibrium conditions satisfied (Figure 9.10). Consumers maximize utility by selecting $\Omega$, the insurance company makes zero profit, and no insurance company can steal away customers without offering a negative-profit contract.

![Figure 9.9](image)

**Figure 9.9.** When only contract $a$ is available, it satisfies the first two conditions of equilibrium. But if contract $\beta$ is offered, purchasers all shift away from $a$, and $\beta$ would earn positive profits. Hence, $a$ fails the third equilibrium condition.

![Figure 9.10](image)

**Figure 9.10.** A stable equilibrium.

Note that the consumer's utility curve is tangent to the zero-profit line at $\Omega$. It can be shown that this condition must hold true for any full-insurance contract — we know the zero-profit line and indifference curves will always be tangent at the full-insurance line for any consumer with any risk level and any set of risk-averse preferences. This tangency at the zero-profit and full-insurance intersection reflects the fact that actuarially fair and full insurance is the most ideal insurance from the consumer’s point of view. And as we have shown, the market with homogeneous customers produces this ideal contract in equilibrium.
9.7 Heterogeneous risk types

So far, we have considered a world where all consumers are identical. We relax this assumption and postulate two risk types in this market: robust and frail.

We assume that these two types actually have a lot in common. Both types of individuals share the same income–utility curve and the same endowment point. However, the robust and the frail have different probabilities of illness \( p \). Let \( p_r \) be the probability of illness for all robust individuals, and let \( p_f > p_r \) be the analogous probability for all the frail individuals. As their names suggest, the robust individuals are less likely than frail individuals to become sick.

Earlier we noted that the slope of the zero-profit line is dependent on profitability of sickness. There will be different zero-profit lines for the robust and for the frail individuals. Because \( p_r < p_f \), the robust zero-profit line is steeper than the frail zero-profit line (see equation (9.1)). Figure 9.11 plots these zero-profit lines in \( I_R - I_I \) space.

There are two types of individuals in this market, so the market-wide probability of falling sick \( \bar{p} = \frac{1}{2} (p_r + p_f) \) is an average of \( p_r \) and \( p_f \), weighted by the fraction of each type in the population. Thus, the slope of the population zero-profit line depends on the composition of the overall population. If there are many frail individuals, for example, the population zero-profit line tends toward the frail zero-profit line. Conversely, if there are many robust individuals, then the population zero-profit line tends toward the robust zero-profit line.

Figure 9.11 depicts a contract \( \delta \) that lies on the population zero-profit line. If the entire population, robust and frail, adopts this contract, then the firm makes zero profit. If robust individuals adopt the contract and the frail refrain, then the firm makes positive profits, since \( \delta \) is to the southwest of the robust zero-profit line. Conversely, if only frail individuals adopt the contract, then the firm makes negative profits, since \( \delta \) is to the northeast of the frail zero-profit line. Whether or not this contract is profitable depends on who takes it up.

9.8 Indifference curves for the robust and the frail

Despite the fact that everyone in the population, robust and frail, has the same income–utility curve, the two types of individuals have different expected incomes and different expected utilities. One consequence of this fact is that each type has its own distinct set of indifference curves in \( I_R - I_I \) space. All risk-averse individuals would be happy to trade income in the healthy state for income in the sick state. But robust individuals are more likely to end up healthy, so they place a relatively higher value on \( I_R \) than do frail individuals.

![Figure 9.12. The indifference curves of the robust individuals \( U_R \) are steeper than the indifference curves of the frail individuals \( U_F \). That steepness means that for an identical gain in sick-state income, robust individuals are willing to sacrifice less healthy-state income than frail individuals are.](image)

Figures 9.12 and 9.13 illustrate how robust and frail individuals value the tradeoff between \( I_R \) and \( I_I \) differently. We plot a pair of sample indifference curves, one for the robust individuals and one for the frail individuals.

Consider an insurance contract that pays out \( x \) in the sick state. How much income in the healthy state would the two types of customers be willing to pay in exchange for receiving \( x \) if sick? The robust individual would be willing to pay at most \( H_R - H_I \) in the healthy state in exchange for \( x \). This tradeoff leaves the robust individuals with the same utility they would receive at \( E \). Because frail individuals value income in the sick state more than the robust individuals, they would be willing to pay up to \( H_F - H_I \), more than robust individuals would.

![Figure 9.13. In the figure, we plot the robust indifference curve that is tangent to the robust zero-profit line on the full-insurance line, as well as the analogous frail indifference curve. These tangency points represent each risk type's own ideal contract.](image)
While Figure 9.12 shows the robust and frail indifference curves that intersect at \( E \), we could conduct a similar analysis at any point in the \( L_p - L_f \) space. In fact, it can be shown that any robust indifference curve crosses every frail indifference curve exactly once, and vice versa. This is called the single-crossing property and is a consequence of the definition of expected utility.

### 9.9 Information asymmetry and the pooling equilibrium

Suppose insurance companies are perfectly able to tell the robust and frail individuals apart, and are legally allowed to exclude certain risk types from certain contracts. Figure 9.14 depicts the stable equilibrium, with each risk type contracting at their respective ideal insurance points, \( \Omega_1 \) and \( \Omega_2 \). Frail customers would prefer to switch from \( \Omega_1 \) to \( \Omega_2 \), because \( \Omega_2 \) yields higher utility, but the insurance company can deny them. If the frail were allowed to switch, they would do so and the insurance company would lose money (\( \Omega_2 \) falls northeast of the population zero-profit line).

![Figure 9.14. Stable equilibrium with heterogeneous customers and perfect information.](image)

In this case where insurers can distinguish robust and frail types, the insurance market functions well. Both risk types purchase their own ideal contract. This is known as the symmetric information equilibrium. This is analogous to the situation in the Akerlof model where cars have heterogeneous quality, but the buyers and sellers have the same symmetric information. Each car is priced according to its own quality, and all cars sell.

What happens if insurance companies, like clueless Akerlofian used-car buyers, cannot tell the difference between robust and frail individuals? In practice, this means insurers cannot restrict any type of person from buying any contract; all offered contracts must be available to everyone, or not offered at all. Insurers are no longer able to forbid the frail type from adopting contracts designed for the robust.

In this case, all the frail customers, previously trapped at \( \Omega_1 \), would disguise themselves as robust and sign up for \( \Omega_2 \). It makes sense for these frail customers to lie about their health — it allows them to get a better deal on insurance. As we have seen, if they do successfully masquerade themselves, the insurance companies make negative profits, so this outcome cannot occur in equilibrium.

From a policy perspective, there is often a desire to have risk types pool together in a single insurance policy, in which both frail and robust types pay the same premium and receive the same payout if sick. This is known as a pooled contract, and an equilibrium consisting of this pooled contract is known as a pooling equilibrium.

**Definition 9.2**

**Pooling equilibrium**: a contract that attracts both robust and frail customers and simultaneously satisfies the equilibrium conditions.

Let us propose a candidate pooling contract \( \alpha \) in Figure 9.15. Any possible pooling equilibrium must rest on the population zero-profit line. If \( \alpha \) lies to the right of the zero-profit line, then the firm loses money. If it lies to the left, then other insurance firms could enter and make money.

![Figure 9.15. A hypothesized pooling equilibrium.](image)

The contract \( \alpha \) satisfies the first condition of equilibrium because both robust and frail types choose \( \alpha \) over \( E \). The second condition is satisfied because both robust and frail types choose \( \alpha \), and it is on the population zero-profit line. Therefore, the firm makes zero profit.

The third condition, however, is not satisfied. Figure 9.16 shows the robust and frail indifference curves that pass through \( \alpha \). Because the curves have different slopes, they form a triangular region that a crafty insurance company can exploit.

![Figure 9.16. The pooling equilibrium is destroyed.](image)
Suppose a different company offered an insurance contract at \( \delta \). Who would it attract? It is above the robust indifference curve that passes through \( \alpha \), so naturally the robust individuals would happily switch to \( \delta \). Meanwhile, the frail individuals choose to stay at \( \alpha \) because their utility level is higher there than at \( \delta \).

Thus, we have a case of adverse selection. The contract \( \delta \) differentially attracts the robust individuals but leaves the frail individuals at \( \alpha \). This is good for the company offering \( \delta \) because the expected payouts to the robust individuals are low, the firm makes positive profits (\( \delta \) is to the southwest of the robust zero-profit line). Meanwhile, the differential selection adversely affects the firm that was offering \( \alpha \). Now only the frail individuals most likely to be sick choose \( \alpha \), which drives up the company’s expected payouts and makes \( \alpha \) unprofitable (\( \alpha \) is to the northeast of the frail zero-profit line).

We picked \( \alpha \) as a candidate pooling equilibrium, but, as we have seen, it is not an equilibrium at all. Our choice of \( \alpha \) was not in any way special – we selected an arbitrary point on the population zero-profit line, which we established as the only place a pooling equilibrium could exist. If we had started with a different candidate point \( \alpha' \) somewhere else on the zero-profit line, the same argument would have held. Because of the relative shapes of the indifference curves, there will always be a triangular zone where an enterprise company can offer a contract like \( \delta \). Thus, no pooling equilibrium can exist.

### 9.10 Finding a separating equilibrium (sometimes)

While a pooling equilibrium can never exist in this model, a different kind of equilibrium, called a separating equilibrium, can sometimes exist. This equilibrium separates the robust and frail types not by discriminating against them directly, but instead by offering different contracts that appeal specifically to each. Since the insurance companies cannot distinguish between the two risk types, this is their only hope of separating them.

**Definition 9.3**

Separating equilibrium: a set of two contracts that satisfies the equilibrium conditions, one that attracts robust customers and one that attracts frail customers.

We have already seen a separating equilibrium under the assumption of perfect information in Figure 9.14. We have also seen that this equilibrium breaks down if the insurance companies cannot distinguish between robust and frail individuals. The equilibrium falls apart specifically when the frail customers flee \( \Omega_2 \) for the much-preferred contract \( \Omega_1 \). This suggests we might be able to create a stable equilibrium if we just make \( \Omega_2 \) less attractive to the frail types – so unattractive that \( \Omega_1 \) looks good by comparison.

Let us try to construct an equilibrium using the strategy described in the previous paragraph. \( \Omega_1 \) belongs in this equilibrium set, but \( \Omega_2 \) cannot be included because it breaks the equilibrium. Consider the frail indifference curve that passes through \( \Omega_2 \) (Figure 9.17). In order to find a second insurance contract \( \Omega_2 \) that will not tempt the frail individuals to leave \( \Omega_1 \), we need to put \( \Omega_2 \) on or below this indifference curve.

Suppose we place \( \Omega_2 \) at the intersection of the indifference curve and the robust zero-profit line. Let us consider whether the pair of contracts \( (\Omega_1, \Omega_2) \) is an equilibrium.

The frail types are indifferent between \( \Omega_1 \) to \( \Omega_2 \), since both lie on a single frail indifference curve. We assume that the frail types choose \( \Omega_1 \) over \( \Omega_2 \) since they are indifferent between the two.\(^2\) At the same time, the robust types prefer \( \Omega_2 \) to \( \Omega_1 \).

It may be counter-intuitive that the frail individuals pass up \( \Omega_2 \), which is actuarially unfair in their favor. The explanation is that, while \( \Omega_1 \) is a lower-price insurance contract, it provides only a small quantity of insurance; it is far from full. The frail types, who are very likely to become ill, prefer \( \Omega_1 \), which is expensive but full insurance.

So, does the pair \( (\Omega_1, \Omega_2) \) satisfy the definition of equilibrium? As we have seen, each risk type maximizes its own utility over the available alternatives; the two risk types voluntarily separate themselves between the two contracts. Because those contracts lie on the risk types respective zero-profit lines, the insurance firm makes zero profit on each contract. The only question is whether another company can enter the market, steal away some or all of the customers, and earn at least zero profit. If a company can do this, the equilibrium breaks down.

Suppose another firm called Adverse-Selection-R-Us enters with a contract in the region labeled \( R_1 \) in Figure 9.17 in an attempt to disrupt the market. The reader should be able to see that any such contract would attract the frail individuals but not the robust ones. Those contracts would therefore be unprofitable for the entering firm, so no such contract would invalidate the proposed equilibrium \( (\Omega_1, \Omega_2) \).

Suppose AS-R-Us instead enters with a contract in region \( R_2 \). Both frail and robust risk types would choose such a contract over their existing ones. However, any such contract would lose money; the entirety of \( R_2 \) is to the northeast of the population zero-profit line. While AS-R-Us makes money off its robust customers with this contract, frequent payouts to the frail customers result in overall negative expected profits.

Increasingly desperate, AS-R-Us turns to region \( R_3 \). Contracts in this region do successfully skim the robust customers away from the original firm. When we considered the pooling equilibrium, this skimming was sufficient to make the contract profitable and break the equilibrium. This is not the case for the proposed separating equilibrium. Because \( \Omega_2 \) is already zero-profit for robust individuals, anything more generous (like the contracts in \( R_3 \)) will be unprofitable for AS-R-Us.

\(^2\) We can assume that the frail types will take \( \Omega_2 \) over \( \Omega_1 \) even though they lie on the same indifference curve. We are allowed to do this because we can always imagine moving \( \Omega_1 \) very slightly to make it strictly inferior. Analytically, it is often simpler to just assume that the frail individuals in question will do what we want when they are actually indifferent between multiple alternatives.
In fact, there is no place where another firm could enter and break the equilibrium. We have shown that $\Omega_1$, $\Omega_2$ fulfills all three equilibrium conditions and separates the frail types; it is thus a separating equilibrium.

However, it is not true that a separating equilibrium is always guaranteed to exist. If there are enough robust types in the population, AS-R-Us could offer a contract that attracts both types and manage to turn a profit. In Figure 9.17, this was impossible because there were not enough robust individuals in the market to offset the losses from the frail individuals for contracts in $\Omega_1$.

Figure 9.18 depicts such a situation where no separating equilibrium exists. The large number of robust individuals in the population rotates the population zero-profit line toward the robust zero-profit line. This creates a new region $\Omega_2$ that did not exist in Figure 9.17. If AS-R-Us enters with a contract in $\Omega_2$, it will attract both types of customers — frail types who are willing to give up full insurance to get lower premiums, and robust types who are willing to give up actuarially fair insurance to get something closer to full coverage. Since contracts in $\Omega_2$ are profitable, the candidate separating equilibrium $(\Omega_1, \Omega_2)$ fails under these circumstances.

The Rothschild–Stiglitz model thus predicts a separating equilibrium only when the robust population is small enough relative to the whole population. Like the Akerlof market, this market is hindered by asymmetric information and can only function under particular circumstances. Even when it does function, it does not perform as well as it could with symmetric information.

So far, we have found two separating equilibria: one when we assumed symmetric information and that insurers could discriminate on the basis of health $(\Omega_1, \Omega_2)$ and one when we assumed asymmetric information $(\Omega_1, \Omega_3)$. Figure 9.19 depicts the two equilibria and the indifference curves that pass through their contracts.

In both the symmetric and asymmetric information cases, the frail types purchase the same insurance contract. One might think that frail types would suffer if insurance companies developed a technique, such as a genetic test, to differentiate them from the robust types. But the Rothschild–Stiglitz model implies that frail types are indifferent to whether insurance companies can distinguish them from the robust.

Instead, it is the robust individuals who have the most at stake. When insurers can distinguish between the two types of customers, they can offer an exclusive contract to the robust that features actuarially fair, full insurance $(\Omega_3)$ — the ideal contract for robust individuals. When insurers cannot discriminate on the basis of risk type, they cannot afford to offer this attractive, ideal contract because the frail individuals will masquerade as robust. So the robust individuals are left with a low-quantity insurance contract $(\Omega_1)$ that falls substantially below full insurance.

In this sense, the frail individuals exert a negative externality on the robust merely by the fact of their existence. Because the potential exists for frail individuals to poison the insurance pool, insurers must severely limit the quantity of insurance available to robust individuals.

9.11 Can markets solve adverse selection?

Suppose that we live in a world where everyone is risk-averse, but different people face different levels of risk. Let us also assume that the bad side-effect of insurance, moral hazard, is at most a minor problem that is outweighed by the benefits of insurance (see Chapter 11). Under these conditions, it is socially optimal for everyone to be fully insured.

If a society wants to ensure that all of its members have full insurance coverage, how would it go about accomplishing that goal? The two models of adverse selection we have studied indicate that adverse selection can sometimes cause markets to unravel or disappear altogether. An insurance market with asymmetric information like the one described in the Rothschild–Stiglitz model has trouble producing a "good" outcome if a society wants full insurance for all. Even in a separating equilibrium where both robust and frail customers can buy welfare-enhancing insurance, robust customers are quantity constrained and cannot insure fully.

If we assume a goal of universal full insurance, there are a few ways it could be accomplished. A government could force everyone to buy an insurance contract at the intersection of the full-insurance line and the population zero-profit line. This would result in discount insurance for the frail but also actuarially unfair insurance for the robust. In practice, most policy attempts focus on this sort of strategy: mandating people of different risk levels to form a single insurance pool. Nationalized health insurance and insurance mandates, which we will read much more about in Chapter 15, both represent attempts to defeat adverse selection by essentially outlawing it.

In the remainder of this chapter, we consider ways in which private markets may be able to solve the adverse selection problem without forcing people to pool against their
The guaranteed renewable contract

Pauly et al. (1995) propose a similar scheme: the guaranteed renewable contract, which is cleverly designed so that both risk types will actually want to remain in the contract of their own volition, and no commitment is required. The guaranteed renewable contract works by frontloading premium payments, shifting premiums into the younger years when Peter and Tim still do not know who will stay healthy and who will be ill.

In the guaranteed renewable contract, the premiums are set to fall gradually over time. By the time Tim is significantly sicker than Peter, both are paying low premiums and neither wants to leave. Effectively, the younger versions of Peter and Tim pay ahead of time for most of the costs that both of them will incur later in life. Rather than having the 50-year-old Peter subsidizing the 50-year-old Tim, the younger version of both subsidize whoever turns out to be sick later in life.

Because at their young age they do not yet know who will be generating those costs, Peter and Tim are both willing to pay high premiums while they are young. Rather than a legally binding commitment, it is these high upfront premiums that effectively lock in customers. Hendel and Lizzieri (2003) find that many life insurance contracts in the US do in fact feature frontloading with premiums that gradually decrease over time relative to mortality risk.

The Cochrane lifetime contract

Cochrane (1995) proposes a similar solution that has the added feature of allowing customers to move between insurers, which fosters greater competition. In his scheme, each insurer offers actuarially fair contracts that are renegotiated yearly. These contracts cover health costs and also provide premium insurance—insurance against higher future premiums (that is, against the risk of becoming frail).

If a customer with a Cochrane lifetime contract is diagnosed with cancer, she not only receives reimbursement for her immediate health care costs but also a lump sum payment covering the higher premiums she will inevitably have to pay in future years. Next year her actuarially fair premiums will be much higher, but she will be able to afford them thanks to her lump sum payment from the previous year. By providing both same-year health insurance and future-year premium insurance, the market becomes much more mobile and competitive. Sick customers are not tied to their current insurers because they have the money to afford actuarially fair premiums anywhere.

9.12 Conclusion

Economic models must provide sharp predictions in order to be testable in the real world and useful in guiding policy. The Rothschild–Stiglitz model makes two such predictions. First, no pooling equilibrium can exist: robust individuals will never voluntarily subsidize frail individuals in an insurance market, even if both types are risk-averse. Insurance markets are well designed to pool risk among people with similar risk profiles, but not across populations with differing risk profiles.

Second, if a separating equilibrium exists, frail individuals will be fully insured, but pay a high premium per unit of coverage. Robust individuals will be partially insured, but pay a lower premium appropriate to their health risk. Insurance companies will never offer bulk discounting—in fact, they offer the opposite.
In the next chapter, we examine the empirical evidence that tests these two predictions in real-world insurance markets. Our discussion will cover chain-smoking teachers, terminal AIDS patients, and frail college professors.

9.13 Exercises

Review the basic assumptions of the Rothschild–Stiglitz model before answering these questions. Many exercises will refer to these basic assumptions.

Comprehension questions

Indicate whether the statement is true or false, and justify your answer. Be sure to state any additional assumptions you may need.

1. In a Rothschild–Stiglitz model with asymmetric information and heterogeneous risk types, the frail population would be worse off if insurance companies were suddenly able to distinguish between the two types of customers, because they could no longer pretend to be healthy.

2. The Rothschild–Stiglitz model predicts that people who own life insurance should have fewer unobserved traits (that is, unobserved by insurance companies) that lead to a higher risk of death when compared against people with the same level of income but who do not own life insurance.

3. In a Rothschild–Stiglitz model separating equilibrium, there is a volume discount for insurance purchases — those who choose to buy more insurance pay a lower per-unit price for it.

4. In a Rothschild–Stiglitz model separating equilibrium, low-risk consumers of insurance are quantity constrained. They cannot buy as much insurance as they want because the insurance company is worried it will lose money on them.

5. Under certain circumstances in the Rothschild–Stiglitz model, a separating equilibrium cannot exist.

6. In the Rothschild–Stiglitz model, an individual who is offered a choice between full insurance and no insurance will always choose full insurance if they are risk-averse.

7. A pooling equilibrium can exist if the contract being offered lies on the same indifference curve as the endowment point of the robust population.

8. Under the typical assumptions of the Rothschild–Stiglitz model, there is nothing that an insurance company can do to distinguish between robust and frail customers.

9. Private markets are powerless to combat adverse selection, so the only solution is a government-mandated insurance contract.

10. The main advantage of a Cochrane insurance contract over a guaranteed renewable contract is that it does not rely on a legally unenforceable binding lifetime commitment.

Analytical problems

11. A medical test that an insurance company could use to distinguish between high- and low-risk types would create an equilibrium in which both high- and low-risk types could have full insurance. Sketch a brief proof using a diagram. Why is this equilibrium not an equilibrium under the normal information asymmetry assumptions? Show which of the three equilibrium criteria does not hold.

12. Consider Figure 9.20.

![Figure 9.20. Candidate separating equilibrium.](image)

a. Explain why the frail-type indifference curve pictured, $U_f$, is not a valid indifference curve.

b. Draw your own version of this figure and label two contracts, $A$ and $B$. Draw the two contracts such that:
   - contract $A$ is strictly better than $B$, with more income in both states of the world, and
   - the customer with the pictured indifference curve nonetheless prefers contract $B$ to contract $A$.

c. Draw a valid version of a frail-type indifference curve that intersects $\Omega_1$, $\Omega_2$, and the full-insurance line.

d. Is $(\Omega_1, \Omega_2)$ a valid separating equilibrium? Defend your answer.

13. A tax on healthy people. Consider the basic Rothschild–Stiglitz model with asymmetric information and robust and frail customers.

a. Suppose the government imposes a Wellness Tax, $\tau > 0$, on robust and frail types but collects on this tax only when they are healthy (that is, there is no tax if they turn out to be sick). Will a separating equilibrium still be possible? Draw a version of the Rothschild–Stiglitz diagram to support your answer.

b. Will a separating equilibrium be possible if the tax $\tau > 0$ is imposed on all customers in both sick and healthy states? Again, support your answer graphically.

14. Review Figure 9.18, which depicts a separating equilibrium breaking down. In this figure, the separating equilibrium breaks down because the zero-profit line is too far to the right. But this is not the only way that a separating equilibrium can fail.

a. Draw a version of the Rothschild–Stiglitz model where a separating equilibrium holds, but just barely (the robust-type indifference curve should almost touch the aggregate zero-profit line).

b. Imagine that all the robust types in the insurance market suddenly become much more risk-averse. How would this change the shape of their indifference curves? Show how this change can unravel the separating equilibrium.

15. The Rothschild–Stiglitz model and its discontents. Imagine policymakers in the fictional nation of Poria are trying to create a Porian National Insurance Program (FNIP) that will bring full insurance to all the citizens of Poria.