DEMAND FOR INSURANCE

Lecture 5
Why buy insurance?

- Demand for insurance driven by the fear of the unknown
  - Hedge against risk -- the possibility of bad outcomes

- Purchasing insurance means forfeiting income in good times to get money in bad times
  - If bad times avoided, then money lost
  - Ex: The individual who buys health insurance but never visits the hospital might have been better off spending that income elsewhere.
Risk aversion

- Hence, risk aversion drives demand for insurance

- We can model risk aversion through utility from income $U(I)$
  - Utility increases with income: $U'(I) > 0$
  - Marginal utility for income is declining: $U''(I) < 0$
Income and utility

- Graphically,
  - Utility increasing with income $U'(I) > 0$
  - Marginal utility decreasing $U''(I) < 0$
Adding uncertainty to the model

- An individual does not know whether she will become sick, but she knows the probability of sickness is $p$ between 0 and 1
  - Probability of sickness is $p$
  - Probability of staying healthy is $1 - p$

- If she gets sick, medical bills and missed work will reduce her income
  - $I_S = \text{income if she does get sick}$
  - $I_H > I_S = \text{income if she remains healthy}$
Expected value

- The **expected value** of a random variable $X$, $E[X]$, is the sum of all the possible outcomes of $X$ weighted by each outcome’s probability.

  - If the outcomes are $x_1, x_2, \ldots, x_n$, and the probabilities for each outcome are $p_1, p_2, \ldots, p_n$ respectively, then:
    \[ E[X] = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n \]

- In our individual’s case, the formula for expected value of income $E[I]$:
  \[ E[I] = p I_S + (1-p) I_H \]
Example: expected value

- Suppose we offer a starving graduate student a choice between two possible options, a lottery and a certain payout:

  **A:** a lottery that awards $500 with probability 0.5 and $0 with probability 0.5.
  **B:** a check for $250 with probability 1.

- The expected value of both the lottery and the certain payout is $250:

  \[ E[I] = \pi I_S + (1-\pi) I_H \]
  \[ E[A] = .5(500) + .5(0) = $250 \]
  \[ E[B] = 1(250) = $250 \]
People prefer certain outcomes

- Studies find that most people prefer certain payouts over uncertain scenarios

- If a student says he prefers uncertain option, what does that imply about his utility function?

- To answer this question, we need to define expected utility for a lottery or uncertain outcome.
Expected Utility

- The expected utility from a random payout $X$ $E[U(X)]$ is the sum of the utility from each of the possible outcomes, weighted by each outcome’s probability.

- If the outcomes are $x_1, x_2, \ldots, x_n$, and the probabilities for each outcome are $p_1, p_2, \ldots, p_n$ respectively, then:
  $$E[U(X)] = p_1 U(x_1) + p_2 U(x_2) + \cdots + p_n U(x_n)$$
The student’s preference for option B over option A implies that his expected utility from B, is greater than his expected utility from A:

\[
E[U(B)] \geq E[U(A)]
\]

\[
U($250) \geq 0.5 \ U($500) + 0.5 \ U($0)
\]

In this case, even though the expected values of both options are equal, the student prefers the certain payout over the less certain one.

This student is acting in a risk-averse manner over the choices available.
Expected utility without insurance

- Lottery scenario similar to case of insurance customer
  - She gains a high income $I_H$ if healthy, and low income $I_S$ if sick.

- Uncertainty about which outcome will happen, though she knows the probability of becoming sick is $p$
  - Expected utility $\mathbb{E}[U(I)]$ is: ???
Expected utility without insurance

- Lottery scenario similar to case of insurance customer
  - She gains a high income $I_H$ if healthy, and low income $I_S$ if sick.

- Uncertainty about which outcome will happen, though she knows the probability of becoming sick is $p$
  - Expected utility $E[U(I)]$ is:
    $$E[U(I)] = p \cdot U(I_S) + (1-p) \cdot U(I_H)$$
Consider a case where the person is sick with certainty \((p = 1)\):
- \(E[U] = U(I_S)\) equals the utility from certain income \(I_S\) (Point S)

Consider case where person has no chance of becoming sick \((p = 0)\):
- \(E[U] = U(I_H)\) equals utility from certain income \(I_H\) (Point H)
What if $p$ lies between 0 and 1?

- For $p$ between 0 and 1, **expected utility** falls on a line segment between S and H.
For $p = 0.25$, person’s expected income is:

$$E[I] = 0.25 \cdot I_S + (1 - 0.25) \cdot I_H$$

Expected Utility at that point is $E[U(I)]$ (Point A)
 Expected utility and expected income

- Crucial distinction between
  - Expected utility $E[U(I)]$
  - Utility from expected income $U(E[I])$

For risk-averse people, $U(E[I]) > E[U(I)]$
Risk-averse individuals

Synonymous definitions of risk-aversion:

- Prefer certain outcomes to uncertain ones with the same expected income.
- Prefers the utility from expected income to the expected utility from uncertain income
  - \( U(E[I]) > E[U(I)] \)
- Concave utility function
  - \( U'(I) > 0 \)
  - \( U''(I) < 0 \)
A basic health insurance contract

- Customer pays an upfront fee
  - Payment $r$ is known as the *insurance premium*
- If ill, customer receives $q$ -- the *insurance payout*
- If healthy, customer receives nothing

- Either way, customer loses the upfront fee
- Customer’s final income is:
  - Sick: $I_s + q - r$
  - Healthy: $I_H + 0 - r$
Income with insurance

- Let $I_H'$ and $I_S'$ be income with insurance
  - Sick: $I_S' = I_S + q - r$
  - Healthy: $I_H' = I_H + 0 - r$

- Remember that risk-averse consumers want to avoid uncertainty
- For them, optimally $I_H' = I_S'$
Full insurance

- Full insurance means no income uncertainty
  \[ I_S' = I_H' \]

- Final income is *state-independent*
  - Regardless of healthy or sick, final income is the same

- Risk-averse individuals prefer **full** insurance to **partial** insurance (given the same price)
State independence implies

\[ I_H' = I_S' \]

So

\[ I_H + o - r = I_S + q - r \]

\[ I_H = I_S + q \]

\[ q = I_H - I_S \]

The payout from a full insurance contract is the difference between incomes without insurance.
Actuarily fair insurance

- Actuarily fair means that insurance is a *fair* bet
  - i.e. the premium equals the expected payout
    \[ r = \text{???} \]

- Insurer makes zero profit/loss from actuarially fair insurance *in expectation*
Actuarially fair insurance

- Actuarially fair means that insurance is a fair bet
  - i.e. the premium equals the expected payout
    \[ r = p \times q \]

- Insurer makes zero profit/loss from actuarially fair insurance in expectation
Actuarially fair, full insurance

Healthy State

\[ I'_H = I_H - r \]

\[ = I_H - pq \]

\[ = I_H - p(I_H - I_S) \]

\[ = pI_S + (1 - p)I_H \]

\[ I'_H = E[I]_p \]

Sick State

\[ I'_S = I_S - r + q \]

\[ = I_S - pq + q \]

\[ = I_S - p(I_H - I_S) + (I_H - I_S) \]

\[ = pI_S + (1 - p)I_H \]

\[ I'_S = E[I]_p \]

Notice consumers with actuarially fair, full insurance achieve their *expected income* with certainty!
Insurance and risk aversion

- As we have seen, simply by reducing uncertainty, insurance can make this risk-averse individual better off.

- Relative to the state of no insurance, with insurance she loses income in the healthy state ($I_H > I'_H$) and gains income in the sick state ($I_S < I'_S$).

- In other words, the risk-averse individual willingly sacrifices some good times in the healthy state to ease the bad times in the sick state.
Now consider the same insurance contract from the point of view of the insurer:

- **Premium** $r$
- **Payout** $q$
- **Probability of sickness** $p$

$$E[\Pi] = \text{Expected profits}$$

$$E[\Pi(p, q, r)] = (1 - p)r + p(r - q)$$

$$= r - pq$$
Fair and unfair insurance

- In a perfectly competitive insurance market, profits will equal zero

\[ E[\Pi(p, q, r)] = 0 \implies r = pq \]

- Same definition as actuarially fair!

- An insurance contract which yields positive profits is called **unfair insurance**:

\[ E[\Pi(p, q, r)] > 0 \implies r > pq \]

- An insurer would never offer a contract with negative profits
Full vs. partial insurance

- Partial insurance does not achieve state-independence

  - Full insurance
    \[ I'_S = I'_H \]
    \[ I_S - r + q = I_H - r \]
    \[ I_S + q = I_H \]
    \[ q = I_H - I_S \]

  - Partial insurance
    \[ I'_S < I'_H \]
    \[ I_S - r + q < I_H - r \]
    \[ I_S + q < I_H \]
    \[ q < I_H - I_S \]

- Size of the payout \( q \) determines the fullness of the contract
  - Closer \( q \) is to \( I_H - I_S \), the fuller the contract
Comparing insurance contracts

- $A^F$ -- Actuarially fair & full
- $A^P$ -- Actuarially fair & partial
- $A$ -- Uninsurance

- $U(A^F) > U(A^P) > U(A)$
The ideal insurance contract

- For anyone risk-averse, actuarially fair & full insurance contract offers the most utility
  - Hence, it is called the ideal insurance contract

- Ideal and non-ideal insurance contracts:

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<th>Fair</th>
<th>Unfair</th>
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<td>Full</td>
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Comparing non-ideal contracts

- $A^F$ – Full but actuarially unfair contract
- $A^P$ – Partial but actuarially fair contract
Comparing non-ideal contracts

- In this case, $U(A^F) > U(A^P)$
  - Even though $A^F$ is actuarially unfair, its relative fullness (i.e. higher payout) makes it more desirable

- But notice if contract $A^F$ became more unfair, then expected income $E[I]$ falls
  - If too unfair, $A^F$ may generate less utility than $A^P$

- Similarly, $A^P$ may become more full by increasing its payout
  - Uncertainty falls, so point $A^P$ moves
  - At some point, this consumer will be indifferent between the two contracts
Comparing non-ideal contracts

- $A^F$ – Insurer Profit: $E[I] - E[I]^F$
- At $F'$: consumer indifferent between $F'$ and $P$
- Max premium that $F'$ can charge is $E[I] - E[I]^F' + p \times (I_H - I_S)$ if plan $P$ is available
Conclusion

- Demand for insurance driven by risk aversion
  - Desire to reduce uncertainty
  - Diminishing marginal utility from income
  - $U(I)$ is concave, so $U''(I) < 0$
  - $U(E[I]) > E[U(I)]$

- Risk aversion can explain not only demand for insurance but can also help explain
  - Large family sizes
  - Portfolio diversification
  - Farmers scattering their crops and land holdings
Fundamental Tradeoff in Insurance

- Zeckhauser’s (1970) Dilemma:
  - Increases in risk protection from insurance increase moral hazard
  - Enhanced ex post incentives necessarily increase exposure to risk
  - 2nd best tradeoff requires an understanding of the welfare benefits and costs of each
  - Insurance for dental vs. inpatient hospital care?