Our running example for this worksheet is the ever-exciting sample space given by the set of outcomes for two coin flips. That is

\[ S = \{HH, HT, TH, TT\} \]

1. Set membership (applies to an element).
   (a) \( \in \) “is an element of.” E.g. \( HH \in S \) “\( HH \) is an element of \( S \).”
   (b) \( \notin \) “is not an element of” E.g. \( HHH \notin S \) “\( HHH \) is not an element of \( S \).”

2. Set relationships (applies to a set).
   (a) \( = \) Set equality. Two sets, \( S \) and \( S' \), are equal if every element of \( S \) is an element of \( S' \) and vice versa.
   (b) \( \subset \) “is contained in” or “is a subset of”

\[ \{HT, TH\} \subset S \]

Note the distinction between subset and element \( \{HH\} \subset S \) but \( HH \notin S \)
   (c) \( \supset \) ‘contains’ or ‘is a superset of’

\[ \{HH, TH, HT\} \supset \{HT, TH\} \]

3. Null set or empty set, \( \emptyset \). Set with no elements.

4. Logical operations
   (a) \( A \cup B \). Union: The union of set \( A \) and set \( B \) is the set of elements in either \( A \) or \( B \).

\[ \{HH, HT, TH\} \cup \{TH, HT, TT\} = S \]

Extended union:

\[ \bigcup_{i=1}^{n} X_i = X_1 \cup X_2 \cup \ldots \cup X_n \]

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1In some contexts, distinctions are drawn between weak and strong versions of these relationship operators. E.g. “\( A \subset B \)” would be read as “\( A \) is a proper subset of \( B \),” or “\( A \) is strictly contained in \( B \)” implying that there is at least one member of \( B \) that is not in \( A \). This could then be distinguished from “\( A \subseteq B \)” or “\( A \) is weakly contained in \( B \),” which remains true even when \( A \) and \( B \) are equal. Most probability books use only the simpler symbol \( \subset \) to denote the more general relationship that remains true when \( A \) and \( B \) are equal.
(b) \( A \cap B \). Intersection: The intersection of \( A \) and \( B \) is the set of elements contained in both \( A \) and \( B \). (note: Some books, Ross in particular, denote \( A \cap B \) as \( AB \))

\[ \{HH, HT, TH\} \cap \{TH, HT, TT\} = \{HT, TH\} \]

Extended intersection:

\[ \bigcap_{i=1}^{n} X_i = X_1 \cap X_2 \cap \ldots \cap X_n \]

(c) Complement. \( \sim \) or \( A^c \): The complement of set \( A \) is everything in the universal set \( S \) (the sample space, in our context) that is not in \( A \).

\[ \sim \{HH, HT, TT\} = \{TH\} \]

Note:

\[ \sim A \cup A = S \]
[\[ \sim A \cap A = \emptyset \]

5. Algebraic rules

(a) Commutative Laws

\[ A \cup B = B \cup A \text{ and } A \cap B = B \cap A \]

(b) Associative Laws

\[ (A \cup B) \cup C = A \cup (B \cup C) \]
\[ (A \cap B) \cap C = A \cap (B \cap C) \]

(c) Distributive Laws

\[ (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \]
\[ (A \cap B) \cup C = (A \cup C) \cap (B \cup C) \]

(d) Demorgan's Laws

\[ \sim (A \cup B) = \sim A \cap \sim B \]
\[ \sim (A \cap B) = \sim A \cup \sim B \]

6. Additional terms and ideas

(a) Mutually exclusive sets

They have an empty intersection. If \( A_i \cap A_j = \emptyset \), for all \( i, j \), then \( \{A_1, A_2, \ldots, A_n\} \) are mutually exclusive.

(b) Partition

A partition divides a set into a set of mutually exclusive and exhaustive subsets.

If \( A_i \cap A_j = \emptyset \), for all \( i, j \) and \( \bigcup_{i=1}^{n} A_i = B \), then \( \{A_1, A_2, \ldots, A_n\} \) are a partition of \( B \).

(c) A 'divide and conquer' rule for sets.

Any set can be broken down into the part of it that intersects with another set and the part that does not. That is,

\[ A = (A \cap B) \cup (A \cap \sim B) \]