Problem 1

There are three newspapers (I, II, III) in a town with 100,000 people. The percentages of townspeople reading each paper is as follows:

- I: 10%  (I and II: 8%  I and II and III: 1%)
- II: 30%  (I and III: 2%)
- III: 5%  (II and III: 4%)

Note that the 10% who read newspaper I includes those who read other newspapers as well; same for newspapers II and III.

(a) How many people read only one newspaper?

(b) How many people read at least two newspapers?

(c) I and III are morning papers and II is an evening paper. How many people read at least one morning paper and one evening paper?

(d) How many people do not read any newspapers?

(e) How many people read only one morning paper and one evening paper?

Problem 2

Suppose that A and B are mutually exclusive events for which \( P(A) = 0.3 \) and \( P(B) = 0.5 \). What is the probability that

(a) either A or B occurs;

(b) A occurs but B does not;

(c) both A and B occur?

(d) Why don't \( P(A) \) and \( P(B) \) sum to one?

Problem 3

The following data are reported in a study of a group of 1000 subscribers to a magazine. In reference to job, marital status and education, there were 312 professionals, 470 married persons, 525 college graduates, 42 professional college graduates, 147 married college graduates, 86 married professionals and 25 married professional college graduates. Show that these numbers must be incorrect.
Problem 4

A community organization consists of 20 families, of which 4 have one child, 8 have two children, 5 have three children, 2 have four children and 1 has five children.

(a) If one of these families is chosen at random, what is the probability that it has \(i\) children, for \(i = 1, 2, 3, 4, 5\)?

(b) If one of the children is chosen at random, what is the that probability that this child comes from a family having \(i\) children, for \(i = 1, 2, 3, 4, 5\)?

Problem 5

(a) If \(P(E) = .9\) and \(P(F) = .8\), show that \(P(E \cap F) \geq .7\).

(b) Show that in general, \(P(E \cap F) \geq P(E) + P(F) - 1\).

Problem 6

Prove that \(P(E \cap F^c) = P(E) - P(E \cap F)\).

Problem 7

Bertsekas and Tsitsiklis, Ch.1 problems 5 and 6. (See the “Handouts and Links” section of the course website if you don’t have a copy of B&T.)

Problem 8

After an election, there are 4 parties holding seats in a legislature. Party A has 40% of the seats, Party B 30%, Parties C and D have 15% each. A majority government will form and it will be a minimum winning coalition (MWC), in the sense that it will not contain more parties than needed for a majority. (Because the whole party will either support the government or not, the MWC may have more than 50% +1 of the seats.)

(a) If each MWC is equally likely, find the following probabilities
   (i) Party A is in government.
   (ii) Party B is in government.
   (iii) Party C is in government.
   (iv) Party D is in government.

(b) Should the probabilities you found in part (a) add up to one? Why or why not?

Problem 9

Another coalition formation problem. There are four parties, with seat shares as follows: Party A has 40%, Party B has 25%, Party C has 19%, Party D has 16%.

(a) One party will be chosen as a “formateur,” and thereby given the chance to form a government coalition. In this country, governments must have the support of majority (at least 50% + 1 of the legislators) and legislators will only support governments that include their own party. Parties want to be in government and they want to share government power with as few other legislators as possible. Thus, the formateur will pick a government coalition that is the smallest
possible majority (in terms of number of legislators, not number of parties) including itself. Each party is equally likely to be chosen as formateur. What is each party’s probability of being in government?

(b) Given the formation process in part (a), what is each party’s probability of being in a government formed by a different party?

(c) Now suppose each individual legislator has an equal chance of being chosen as formateur. Everything else is the same as in part (a) - the formateur will always include her own party, and whatever other parties she needs to construct the smallest possible coalition that is still a majority. What is each party’s probability of being in government?

(d) Given the formation process in part (c), what is each party’s probability of being in a government formed by a different party?