Set Theory Rules and Concepts

Continuing the example sample space $S = \{HH, HT, TH, TT\}$

1. Algebraic rules
   
   (a) Commutative Laws: $A \cup B = B \cup A$ and $A \cap B = B \cap A$
   
   (b) Associative Laws: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$
   
   (c) Distributive Laws: $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
   
   (d) Demorgan’s Laws: $(A \cap B)^c = A^c \cup B^c$ and $(A \cup B)^c = A^c \cap B^c$

   Examples
   
   - $(A \cap B)^c = A^c \cup B^c$. Suppose you are doing electoral canvassing, knocking on doors trying to get out the vote in support of a particular candidate. You want to knock on the doors of people who have not already voted (event $A$) and are likely to support your candidate ($B$). You can skip $(A \cap B)^c$ houses where either they’ve already voted ($A^c$) or they’re not likely to support your candidate ($B^c$).
   
   - $(A \cup B)^c = A^c \cap B^c$. Now suppose you are an organized group, trying to decide which candidates you might interview for a potential endorsement. You want to interview anyone who has a chance of winning ($A$) or anyone who is already an ally of your organization. You can decline to interview $((A \cup B)^c$ people candidates who have no chance of winning ($A^c$) and are not existing allies ($B^c$).

2. Additional terms and ideas
   
   (a) Mutually exclusive sets
   
   They have an empty intersection. If $A_i \cap A_j = \emptyset$, for all $i,j$, then $\{A_1, A_2, ... A_n\}$ are mutually exclusive.
   
   (b) Partition
   
   A partition divides a set into a set of mutually exclusive and exhaustive subsets.

   If $A_i \cap A_j = \emptyset$, for all $i,j$ and $\bigcup_{i=1}^{n} A_i = B$, then $\{A_1, A_2, ... A_n\}$ are a partition of $B$.

   Returning to our government formation model, we can partition set of governments by type: MWC, universal, minority

   But not by party: Governments containing A, governments containing B, governments containing C are not mutually exclusive.

   (c) A ‘divide and conquer’ rule for sets.

   Any set can be broken down into the part of it that intersects with another set and the part that does not. That is, $A = (A \cap B) \cup (A \cap B^c)$

   In order to use probability in probability models we have to keep track of events.

   Problems often come from bad logic in models more than misunderstanding probability per se.

   Venn diagrams helpful for managing set theory issues. They can also be kind of fun (see below.)
If you like this kind of thing, or you just want some practice, here are some questions.

(a) What claims are being made about each of the individuals in terms of their characteristics? E.g. Willie Nelson is said to be a Picker and a Toker but Not a Grinner, Not a Lover, Not a Sinner, Not a Joker and Not a Smoker.

(b) What combinations of characteristics do the unlabeled sets describe. E.g. just above the 7-way intersection that includes Steve Miller is a small triangle that would contain people who Smokers, Tokers, Pickers, Grinners, Lovers and Sinners but Not Jokers.