Bayes’ Law Example

Let’s put some rough numbers in our example of testing positive for a disease. As in the previous lesson, let \( P \) be the event that the person tested positive and \( D \) the event that the person has the disease.

- Suppose that the baseline incidence of the disease is around 0.5%, so that \( \Pr(D) = 0.005 \). That’s slightly higher than (but in the ballpark of) the most common forms of cancer in the most vulnerable age groups. We’d expect one case per 200 people.

- Assume a sensitivity of 95%, meaning \( \Pr(P|D) = 0.95 \). The false negative rate, \( \Pr(P^c|D) \), is thus 0.05.

- Assume specificity of 99%, meaning the “true negative rate” \( \Pr(P^c|D^c) = 0.99 \) and the false positive rate, \( \Pr(P|D^c) = 0.01 \). This is a very good test.

Our question is, given that the test was positive, how likely is the person to have the disease? We want

\[
\Pr(D|P) = \frac{\Pr(P|D) \Pr(D)}{\Pr(P|D) \Pr(D) + \Pr(P|D^c) \Pr(D^c)}
\]

Plugging in these values gives us

\[
\Pr(D|P) = \frac{0.95 \times 0.005}{0.95 \times 0.005 + 0.01 \times 0.995} = 0.323
\]

That is, even though the test is quite accurate, the probability that person who tests positive does not have the disease is fairly high \( \Pr(D^c|P) = 0.677 \).

- People tend to confuse the probability that the test inaccurately gives a positive result (the false positive rate \( \Pr(P|D^c) \)) with the probability that a given positive result is inaccurate \( (\Pr(D^c|P)) \).

- In this example, the probability that a healthy person gets a positive test result \( (\Pr(P|D^c) = 0.01) \) is low compared to the probability that a person with a positive result is actually healthy \( (\Pr(D^c|P) = 0.677) \).

It makes a difference what is conditional on what!

The reason the numbers are so different reflects the fact that the baseline probability of the disease is low.

- Before we knew about the positive test result, the probability this person has the disease was \( \Pr(D) = 0.005 \). In Bayesian lingo, we call this the “prior probability.”

- When we learned about the test, we use Bayes Law to “update our priors.” The updated (or “posterior”) probability \( \Pr(D|P) \) is much higher.

The positive test changed our beliefs, even though we are not just accepting it at face value.